Universidade Federal de Pernambuco
Centro de Tecnologia e Geociências
Programa de Pós-graduação em Engenharia Elétrica

Geraldo Andrade de Oliveira

The Energy-Water Nexus

Recife, Julho de 2014.

Geraldo Andrade de Oliveira

The Energy-Water Nexus

Tese submetida ao Programa de Pós-Graduação em Engenharia Elétrica da Universidade Federal de Pernambuco como parte dos requisitos para obtenção do grau de **Doutor em Engenharia Elétrica**

Orientador: Prof. Fernando Menezes Campello de Souza, Ph.D.

Recife, Julho de 2014.

Catalogação na fonte
Bibliotecária Maria Luiza de Moura Ferreira, CRB-4 / 1469

O48e Oliveira, Geraldo Andrade de.
The Energy-water Nexus / Geraldo Andrade de Oliveira.
- Recife: O Autor, 2014.
96 folhas, il.

Orientador: Prof. Fernando Menezes Campello de Souza, Ph.D.
Tese (Doutorado) – Universidade Federal de Pernambuco. CTG. Programa de Pós- graduação em Engenharia Elétrica, 2014.
Inclui Referências.

1. Engenharia Elétrica. 2. Controle ótimo. 3. Modelo de crescimento econômico. 4. Princípio do Máximo de Pontryagin. 5. Política energética. 6. Cadeia de suprimentos de Leontief. I. Souza, Fernando Menezes Campello de (Orientador). II. Título.

621.3 CDD (22. ed.) UFPE/BCTG/2015-09

Universidade Federal de Pernambuco
Pós-Graduação em Engenharia Elétrica

PARECER DA COMISSÃO EXAMINADORA DE DEFESA DE TESE DE DOUTORADO

GERALDO ANDRADE DE OLIVEIRA

TÍTULO

"ENERGY-WATER NEXUS"

A comissão examinadora composta pelos professores: FERNANDO MENEZES CAMPELLO DE SOUZA, DES/UFPE; RENATO JOSÉ DE SOBRAL CINTRA, DE/UFPE; HÉLIO MAGALHÃES DE OLIVEIRA, DES/UFPE; MÁRCIA MARIA GUEDES ALCOFORADO DE MORAES, DECON/UFPE e ANDRÉ MARQUES CAVALCANTI, CA/UFPE sob a presidência do primeiro, consideram o candidato **GERALDO ANDRADE DE OLIVEIRA APROVADO.**

Recife, 29 de julho de 2014.

CECILIO JOSÉ LINS PIMENTEL
Coordenador do PPGEE

FERNANDO MENEZES CAMPELLO DE SOUZA
Orientador e Membro Titular Externo

HÉLIO MAGALHÃES DE OLIVEIRA
Membro Titular Externo

RENATO JOSÉ DE SOBRAL CINTRA
Membro Titular Interno

MÁRCIA MARIA GUEDES ALCOFORADO DE MORAES
Membro Titular Externo

ANDRÉ MARQUES CAVALCANTI
Membro Titular Externo

Agradecimentos

Agradeço primeiramente a Deus, por ter guiado meu destino tão bem e já me oportunizado tantas situações boas em minha vida.

Sou honrosamente grato ao meu pai, por ter servido de exemplo como homem, profissional, pai, amigo e conselheiro, sem o qual eu não seria quem sou hoje. E como segundo exemplo pra mim, tão importante quanto, agradeço a minha mãe, que por ser uma educadora devotada, bem acima da média dos professores que tive em minha vida, com extrema dedicação à melhoria da relação ensino-aprendizagem, exerceu uma influencia fundamental na minha decisão de me tornar um profissional da área de educação.

Agradeço à minha amada e tão querida esposa Elen, que além de estar prontamente ao meu lado nos momentos bons e ruins, é uma pessoa que assegura minha paz e harmonia, preenchendo minha vida com seu amor, apoio diário, paciência, conselhos e incentivos cotidianamente. Ela, sem duvida, foi um presente escolhido com carinho por Deus para mim.

Sou especialmente grato ao meu orientador, o Professor Fernando Menezes Campello de Souza, que além de um ser humano extraordinário é um acadêmico de referência, onde ao mesmo tempo em que possui uma imensurável amplitude de conhecimento, mantém a simplicidade o suficiente para deixar qualquer interlocutor confortável. No nosso primeiro contato, quando ainda estava sendo estudada a possibilidade dele me orientar, fiquei a vontade o suficiente para confessar meu desejo pessoal da época de migrar de uma carreira profissional em multinacionais já consolidada para a minha atual carreira acadêmica! Enfim, graças ao apoio e capacidade do professor, orientador, conselheiro e amigo Fernando, consegui concluir meu ciclo básico de formação em Mestrados e Doutorado Acadêmicos.

Aos meus familiares e amigos, tanto os que já existiam antes do início deste ciclo acadêmico, como os que vieram durante o processo, deixo meu agradecimento e registro de que sem o apoio positivista deles, seja através do convívio em ambiente de entretenimento, ou no prazer de compartilhar ideias e ideais, a conclusão destas conquistas não seria possível.

E por fim, agradeço aos meus alunos, a quem considero como o meu combustível motivacional e com os quais aprendo a cada dia. Eles também são o elemento chave para o novo ciclo que agora se inicia através da pesquisa e do desenvolvimento dos trabalhos que realizarei.

Geraldo Andrade de Oliveira

Universidade Federal de Pernambuco

21 de Julho de 2014

Resumo da Tese apresentada à UFPE como parte dos requisitos necessários para a obtenção do grau de Doutor em Engenharia Elétrica

The Energy-Water Nexus

Geraldo Andrade de Oliveira

Julho/2014

Orientador: Prof. Fernando Menezes Campello de Souza, Ph.D.
Área de Concentração: Processamento de Energia
Palavras-chaves: Controle Ótimo, Modelo de Crescimento Econômico, Princípio do Máximo de Pontryagin, Politica Energérica, Cadeia de Suprimentos de Leontief
Número de páginas: 96

Produção e consumo de energia e água estão intimamente ligados. Energia e água são recursos valiosos que sustentam a prosperidade humana, e são em grande parte interdependentes. Há exigências e restrições em diversas áreas, como resultado do crescimento econômico e populacional e das mudanças climáticas, que amplificam a vulnerabilidade mútua de energia e água. Compreender a intrínseca relação entre energia e água e desenvolver tecnologias para manter essa relação adequada, é uma medida importante para um futuro sustentável e seguro para qualquer país. Propõe-se considerar ambos os recursos como um só. São Popostos dois modelos para analisar o problema central na relação água-energia: um através de Modelo Dinâmico de Entrada-Saída com Leontief, e outro por um modelo de Controle Ótimo Baseado no Princípio de Máximo de Pontryagin. No Modelo Dinâmico de Entrada-Saída com Leontief, chega-se ao importante resultado de que as variáveis água e energia, quando analisadas em conjunto, apresentam menor erro do que quando analisadas separadamente. Já no modelo de Controle Ótimo Baseado no Princípio de Máximo de Pontryagin, onde os critérios para maximização dos resultados são elencados, vinte e um resultados importantes são apresentados.

Abstract of Thesis presented to UFPE as a partial fulfillment of the requirements for the degree of Doctor in Electrical Engineering

The Energy-Water Nexus

Geraldo Andrade de Oliveira

July/2014

Supervisor: Prof. Fernando Menezes Campello de Souza, Ph.D.
Area of Concentration: Energy Processing
Keywords: Optimal Control, Economy Grouth Model, Pontryagin Maximum Principle, Energy Policy, Leontief Supply Chain
Number of pages: 96

Production and consumption of energy and water are closely intertwined. Energy and water are valuable resources that sustain human prosperity and are largely interdependent. There are demands and restrictions in many areas as a result of economic and population growth and climate change, which amplify the mutual vulnerability of energy and water. Understanding the intricate relationship between energy and water and developing technologies to keep that relationship in balance is an important key to a sustainable and secure future for any country. The time has come to consider both issues as one. Two models are proposed to analyze the central problem in the energy-water nexus: one as an Input-Output Leontief Dynamical Model, and the second by a Pontryagin Maximum Principle Optimal Control Model. In the Input-Output Leontief Dynamical Model, we obtain the important result that water and energy variables, when taken together, yield fewer errors than when analyzed separately. In the Pontryagin Maximum Principle Optimal Control Model, where the criteria for maximizing the results are listed, twenty one important results are presented.

List of Figures

Sumary

Chapter 1

Introduction

Energy and water are of utmost importance for any country's economy and way of life. Understanding the intricate relationship between energy and water and developing technologies to keep that relationship in balance is an important key to a sustainable and secure future for any country. There are tradeoffs between energy and water. Large scale power plants —– nuclear, coal, biomass and of course, hydroelectric —– use lots of water. Conversely, making drinkable, potable water, and piping it into big cities, involving typically large distances certainly requires plenty of energy. Water and energy are strongly tied and certainly dependent on each other, with each affecting the other's availability. Water is necessary for energy development and generation, and energy is needed to supply, use, and treat drinking water and waste water. Both, energy and water are essential to our health, quality of life, and economic growth, and demand for both these resources continues to rise. Water and energy are the two most fundamental resources of modern civilization. People die when they cannot grow food, if water is not available. Without energy, one cannot run computers or power homes, schools, offices, farms, or industrial plants. As the world's population grows in number and affluence, demand for both resources is increasing faster than ever. The Earth holds about eight million cubic miles of fresh water —– tens of thousands of times more than humans' annual consumption. Only about 2.5% of the world's water is fresh water. Unfortunately, less than 1% is accessible via surface sources and aquifers, and the rest is imprisoned in underground reservoirs and in permanent ice and snow cover. Also, the available water is often not clean or located far from population centers. The reality that one of these precious resources, energy or water, might soon impede the use of the other has been under-estimated.

To generate energy, massive quantities of water are consumed, and to deliver clean water, massive quantities of energy are consumed. Desalination, a process that removes salt from water, is the most energy-intensive and expensive option for treating water and is used where alternatives are very limited. Other energy needs associated with water occur at the end-use point, often in households, primarily for heating water, cooling water, washing clothes, and pumping water. Many people are concerned about the perils of peak oil —– running out of cheap oil. A few are voicing concerns about peak water. But almost no one is addressing the conflict between the two: water restrictions are hampering solutions for generating more energy, and energy problems, particularly rising prices, are curtailing efforts to supply more clean water. Physical constraints on the availability of water for the energy sector involve both quantity and quality issues: there may not be enough of it or that which is available may be of poor quality. These restrictions may be natural or may arise from regulation of water use. One cannot build more hydroelectric power plants without taking into account that they impinge on the supply of fresh water. And one cannot build more water delivery and cleaning facilities without increasing demand for energy. Solving the dilemma requires new global policies that integrate energy and water solutions and innovative technologies that help to boost one resource without draining the other. One needs an analytical tool, a mathematical tool, to address this problem. A dynamic mathematical model is proposed here. Water and energy are fundamentally linked. Policy reforms in both industries, however, do not appear to acknowledge the links nor consider their wider implications. This is clearly unhelpful, particularly as policy makers attempt to develop effective responses to water and energy issues, underpinned by prevailing drought conditions and impending climate change, and an ever increasing demand for both resources. Energy production requires a reliable, abundant, and predictable source of water, a resource that is already in short supply throughout much of the world. The time has come to consider both issues as one. Instead of water planners assuming they will have all the energy they need and energy planners assuming they will have all the water they need, they must get in the same room to make decisions. A restructuring of the institutional arrangements is in order. An analytical tool is required. This is the underlying idea of this proposed analytical model. Here the links between the water-energy nexus or the energy-water nexus will be analyzed— in the general context.

1.1 Objectives

The goals of this research were to investigate the energy-water nexus, and analyze the tendencies of important decision making for economic and population growth.

1.1.1 General Objective:

Establish a rationale for better management of the energy-water nexus

1.1.2 Specifics Objective:

- Take a deep breath, review and establish the context of the problem;
- Construct a model adaptable for local context solutions (Supply Chain Model);
- Propose an optimal control model for general context solutions (Optimal Control Model).

1.2 Thesis Structure

This document is divided into five chapters:

- The Problem: Energy and Water Issues (Chapter 2) – Provides a background on water, energy, their interconnection, and previous work on modeling future impacts. Review and have some examples about the energy-water nexus.
- Energy-Water Nexus: An Input-Output Dynamical Model (Chapter 3) – Supply chain management is an essential tool in business administration. The input-output matrix, which is part of an underlying mathematical model, is consolidated using the Leontief Model to analyze the dynamics of the energy-water nexus.
- Energy-Water Nexus: An Optimal Control Model (Chapter 4) – An energy-water nexus mathematical model is proposed, formulated in terms of an optimal control problem representing an evolving economy; an optimal economic growth model. It is written as a maximization of a time-driven social welfare function, subject to constraints defined by income and investment identities, production technologies, the dynamic consumption of reserves, as well as the energy balance and the labor force balance.
- Conclusion and Topics for the Future (Chapter 5) – Conclusion and Topics for the Future are presented.

CHAPTER 2

The Problem: Energy and Water Issues

The general context and issues regarding emerging problems in the energy-water nexus are presented.

2.1 The importance of mathematical modeling for science, society and scarce resources

The dynamics, as stated by Gray and Hotelling, should always be dealt with in continuing growth models that work with exhaustible resources. The modeling, especially in social sciences, which addresses very sensitive issues, opens a wide range of discussions of the peculiarities of each party in the equation composing these models. To understand the plethora of academic discussions, it is necessary to be familiar with a typical optimal control problem. In general, there are three main tools for dynamical systems modeling. They are: dynamic programming, the calculus of variations and optimal control theory. Dynamic programming deals with large applications, but its simplest use is in discrete time problems. The principles of dynamic programming have been developed by the American mathematician Richard Bellman in 1957. Dynamic programming, when dealing with continuous-time problems, involves very advanced mathematical procedures; in particular, it uses partial differential equations to obtain a solution, which makes the process to obtain the solutions to the problems very hard. The calculus of variations was born in the seventeenth century, and had as one of its pioneers, Isaac Newton, who solved a problem of the calculus of variations, and in 1687 published the book *Principia Matematica Philosophiae Naturalis.* The mathematicians, Leibniz, John Bernoulli and James Bernoulli also solved problems similar to Newton's. The calculus of variations is still used, however, and the continued study of variation problems led to development of a more modern method, called the optimal control theory. The optimal control theory is a modern generalization of the calculus of variations that comes from the work of the Russian mathematician, Lev Semenovich Pontryagin, who received a science prize for his "Pontryagin Maximum Principle" [68]. At 14 years of age Pontryagin suffered an accident that left him blind for life. He worked with topology, proving in 1934, one of the problems posed by Hilbert in 1900. Pontryagin in 1952 changed completely the direction of his research. He began studying applied math problems, in particular studying differential equations and control theory. In 1962 he published "The Mathematical Theory of Optimal Processes" with other authors, and his work nowadays is the most significant development of optimal control theory [69]. The application of optimal control theory in the study of economics led to various results, theoretical and practical, which previously could not be formulated. The capital theory, formulated before the calculus of variations, had such a large input from the Pontryagin approach, that it changed what was conventionally called the theory of growth, using the maximum fundamental theorem for optimal control theory, and therefore the theory of growth itself [70]. After this, the use of optimal control theory has become frequent in economic analysis.

As an example, a typical problem of optimal control theory has the following general form: an objective functional that should be maximized; equations that restrict the objective function which could be differential equations (which govern the movements of the system. These determine where the system is, and in what state; the group of dynamic variables that compose these equations are called system state variables) or equality equations (which define some known concept of the problem or some balance that must be improved by the system); and the conditions of transversality (boundary conditions, i.e., initial and terminal conditions). In addition, control variables must be specified in order to highlight which variables must be manipulated by the agent through standards or policies. The objective function must account for the results obtained by the agent request. As the name suggests, it should reflect the main objective of the agent. The restrictions set a workable framework through which must pass the values of the objective function; it delimits the space of actions agent. The transversality conditions provide information from the beginning point to where and how you want to go; these conditions may be restrictive or not regarding the objectives of the agent. The Maximum Principle transforms the problem from its original format to a problem which is equivalent mathematically and easier to be solved. This requires defining the state variables by simple inspection of the original equations; and defining the control variables, which is an individual choice from the proposer modeling the phenomenon. The application of the Maximum Principle reveals the optimal conditions through the derivations of the new objective function and other transversality equivalent conditions to the original interaction. A good economic interpretation of Maximum Principle applied to the economy, to one control variable and one state variable, is exposed in Dorfman (1969)[70]. To see a more complete picture of the three tools, see [68].

2.2 Water Overview

Water has several features that make the public sector play a more essential role in development and management than for other commodities, which can be manipulated efficiently by market structures[71]. Water is usually a liquid, physically speaking. That is why it´s mobile: water tends to flow and evaporate, infiltrate as it moves through the water cycle. This causes problems in mobility identification and measurement of specific units of the resource. The supply of water tends, due to natural climate fluctuations, to be variable, so that the risks of scarcity and excess are one of the biggest problems of water management[72]. The global water supply is fixed overall though, but this value varies locally and in extreme cases can change, leaving a specific location that had a certain supply of water, without water. That type of attribute should be taken into account when talking about complex water supply and distributions projects.

Economically, water can be treated as a natural resource with a high cost of exclusion (when the service is provided to a user, it is difficult to exclude other suppliers), i.e., the exclusive property rights that are the foundations of a market or economic exchange are relatively difficult and costly to be established or enforced. Thus, property rights regarding water are most often incomplete or absent [71, 72]. On the demand side, water is a multiple use resource. These uses are in the best of times competitive, involving exchanges (tradeoffs) between their uses. In the meantime, assuming that there is a manager for the resource, or at least allocations of greater supply to demand, it is possible to study how best to treat this competition.

2.2.1 Water Demand: Considerations Regarding General Water Consumption

The Central Problem is that water is a plentiful resource, but it is not always available for human use in sufficient quantities or at the quality, time and place required. Only about 2.5% of the world's water is fresh water. Of that, less than 1% is accessible via surface sources and aquifers – the rest is locked up in glaciers and ice caps, or is deep underground[43].

This means rooms lighting, powering computers and televisions, and running appliances requires more water on average than the total amount we use in our homes washing dishes and clothes, bathing, flushing toilets and watering lawns and gardens. This huge volume of water has to come from somewhere. Across the country, the agricultural water demand is competing with the pressure of growing populations and other necessities requiring stretching water resources, especially during droughts and heat waves.

The extent to which water resources are under pressure in a particular country or region depends on how human consumption relates to supply. Globally, agriculture is the main user of water, accounting for 70% of water use, followed by industry (including mining and power generation) 19%, and municipal networks that meet the water needs of public and private users, 11% [41–43]. Water resources may vary significantly from one river basin to another and can be located far away from areas where demand is greatest.

Figure 2.1 illustrates the world water resources and consumption:

2.1

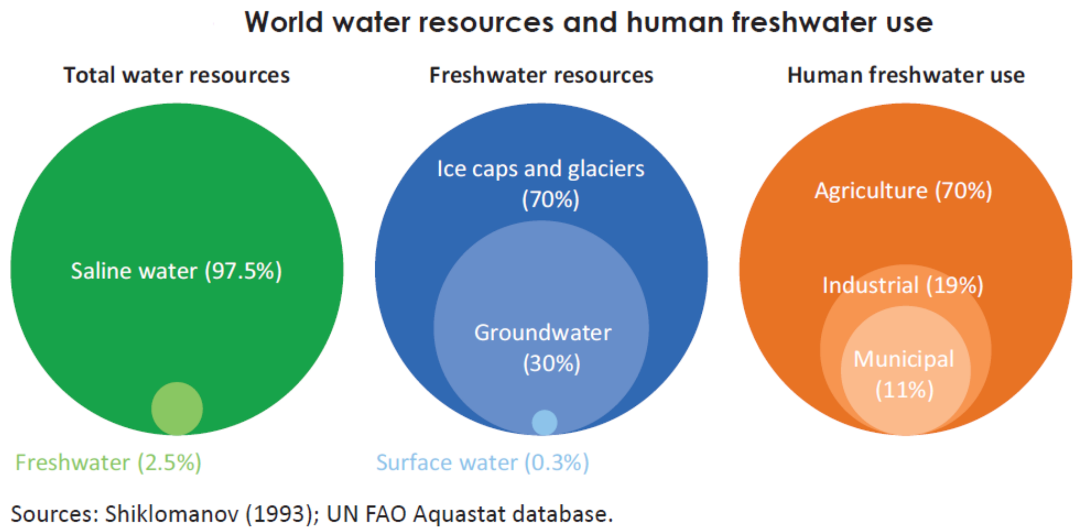

Figure 2.1: The World Water Resources and Consumption

Climate change, along with population and economic growth provide a future with more water restrictions in many regions. Water cycles and climate are closely linked: rising temperatures will accelerate the movement of water, increasing evaporation and precipitation. Expected impacts include falling average flow of surface water (glacier melt being an exception); higher temperatures of the surface water; reduced snow cover and changed timing of spring defrost; a rising sea level, which will contaminate freshwater sources; and droughts, heat waves and floods will become more frequent and more severe [44]. People are changing their diet habits. As they begin to live in urban centers, have easy access to water, they use more than when they were rural residents. And while country people consume more vegetables, in urban centers the demand for meat increases. And the amount of water to produce 1kg of meat is huge (well above many vegetable crops)! Because of the change of diet to increased consumption of meat, people will be indirectly increasing their demand for water. [39].

Among technical alternatives for water supply and management in the Brazilian Northeast one may cite *Manual dos Pequenos Açudes* [93].

2.2.2 Water Demand: Considerations About Primary Energy Production

Water requirements for the production of fossil fuels — including resource extraction, processing and transport of the fuel cycle — vary widely. Conventional natural gas involves the use of water for drilling and minimal processing and is usually much lower than the production of biofuels or other fossil fuels which involve intensive use of water. Shale gas extraction uses additional water for hydraulic fracturing, a well augmentation technique that pumps fluids (water and sand mixed with chemical additives that help the process) into shale formations at high pressure to crack the rock and release the gas. The water requirements for shale gas recovery depend on the gas, the number of hydraulic fracturing treatments performed and the use of water recycling technologies. These factors vary from well to well, but may need to involve rates many times greater than conventional water/gas use. In addition, there is public concern about potential risks of water contamination associated with the development of shale gas, specifically the seepage of fracturing fluids, hydrocarbons or saline water into groundwater and the handling and disposal of wastewater. These risks, which are also present in the development of conventional oil and gas, may be responsibly addressed by a small additional cost, using existing technologies and best practices [45]. Coal production uses water primarily for mining activities such as coal cutting and dust suppression. The amount of water needed depends on the characteristics of the mine, i.e., whether it is on the surface or underground and the requirements for processing and transport. Dry coal increases its quality, but involves additional water. Washing is currently performed mostly just for types of export quality coal, but there is not much space to practice becoming more widespread because of their potential to increase the efficiency of the power plant, as in India. The main concerns of water quality for the production of coal are the disposal of mine tailings and that can pollute surface and groundwater operations. The amount of water needed for oil extraction is determined by the recovery technology in relation to the geology of the oil field and its production history. Water needs for the extraction of conventional oil are relatively minor, similar to conventional gas. Secondary recovery techniques using water flooding to increase the pressure of the reservoir may need to be about ten times higher than those associated with primary recovery, which is based on natural mechanisms to support water. Production of synthetic crude from oil sands is comparatively more water intensive, while in-situ recovery uses, on average, less than one quarter of the amount of water used in surface mining. Refining of crude oil into end use products requires more water for cooling and chemical processes; the amount varies according to technologies used (a cooling system, for example) and the process configuration [43, 47, 48].

Biofuels require water for irrigating crops of raw materials and fuel conversion. Irrigation needs can vary greatly, depending on the crop, the region where it is grown and the efficiency of irrigation technologies. Cultures growing raw materials that require minimal water or that grow them in an area that receives ample rainfall can greatly reduce or eliminate the need for water for irrigation. Rain fed crops grown in Brazil and Southeast Asia, for example, typically make lower demands on water resources than those grown in parts of the United States, where crops are irrigated. Advanced biofuels derived from waste require little or no water for their growth as raw fuel, and water used for these crops is allocated to an activity of primary value (food production, for example); the water use is greater for advanced biofuels crops. [46, 49, 51].

2.2.3 Water Demand: How Power Plants Use Water

How much water a power plant uses depends mainly on which of three basic cooling technologies it uses. "Once-through" systems – which, as the name implies, uses cooling water once before discharging it – This technology requires much more water from sources such as lakes or rivers than other types of cooling systems.

"Recirculation" cooling systems require a fraction of the water that once-through systems do. However, recirculation systems can consume twice as much water as once-through systems, or even more, because the former evaporate much of the cooling water to condense steam.

Dry-cooled systems, which blow air across steam carrying pipes to cool them, use almost no water. However, dry-cooled plants become considerably less efficient when ambient air temperatures are high.

Both recirculation and dry-cooling systems require more energy than once-through systems. Because of that energy penalty, and efficiency losses at high ambient air temperatures, some power plants rely on hybrid cooling systems. These systems—some combination of the aforementioned technologies—operate in dry-cooling mode much of the time, but switch to wet-cooling mode during hot weather [27].

The water demand of power plants varies widely. A nuclear power plant with once-through cooling, for instance, takes in 25,000 to 60,000 gallons of water for each megawatt-hour of electricity it produces, but consumes 100 to 400 gallons. A nuclear plant using recirculated cooling water, on the other hand, takes in 800 to 2,600 gallons per megawatt-hour but consumes 600 to 800 gallons – roughly half the amount withdrawn [38].

Renewable power plants have a wide range of water intensities: low-carbon electricity does not always mean low-water use. Wind turbines – the most widely deployed renewable electricity technology in the United States, aside from hydropower – use essentially no water [27]. The same is true of photovoltaic panels. On the other hand, they rely on recirculation cooling systems. Geothermal, biomass, and some types of concentration solar power plants – all of which use steam to drive turbines – use water in the same range as nuclear or coal plants. Some renewable energy power plants with turbines employ dry cooling, and those require minimal amounts of water.

2.2.4 Water Demand: About the Food and Transport Problem

Food, Transport, Water and Energy

It may not seem so but the systems that help produce, bring and transport fresh food and energy as well as clean, abundant water to all of us, are interconnected. It takes water to create food and energy, it takes energy to move and treat water and to produce food, it takes water and energy to provide transport of these inputs, and sometimes it uses food as a source of energy. These systems have become increasingly more complex and dependent upon one another. As a result, a disturbance in one system can wreak havoc in the others, so it's important to achieve a sustainable balance among them.

Energy, Water and Agriculture

Energy has always been essential for the production of food. Prior to the industrial revolution, the primary energy source for agriculture was the sun; photosynthesis enabled plants to grow, and plants served as food for livestock, which provided fertilizer (manure) and muscle power for farming. However, as a result of the industrialization and consolidation of agriculture, food production has become increasingly dependent on energy derived from fossil fuels. Industrial agriculture is incredibly water intensive. This scarce resource is used for crop irrigation, which accounts for 31% of all water withdrawals in the US, waste management (i.e., for flushing manure out of industrial livestock facilities) and as drinking water for animals. This overuse of water has implications in the energy sector as well. Fresh water is literally the lifeblood of agriculture. We are leaning more about how agriculture uses and impacts water, the large water footprint of food and ways to protect water by understanding the impacts of our food choices and water-smart agriculture practices. As contextualized in the Energy-Water Nexus, pumping, treating and moving such large volumes of water require a great deal of energy.

Modern agriculture relies upon machinery that runs on gasoline and diesel fuel (e.g., tractors and combines), and equipment that uses electricity (e.g., lights, pumps, fans, etc.). Much of the food produced today is highly processed and heavily packaged, which further increases its energy footprint. As a result of consolidation and centralization of production, foods are often transported long distances, requiring additional energy inputs.

Energy policy also affects agriculture. For example, world energy demands require the production of billions of gallons of ethanol, which is primarily and controversially derived from biomass such as corn. Corn grown for ethanol takes land away from food production and, in states where corn is irrigated, uses a significant amount of water.

Given the growing population food requirements, the world's finite supply of fossil fuels and the adverse environmental impacts of using this nonrenewable resource, the existing relationship between agriculture and energy must be dramatically altered. Among the most obvious solutions is to simply improve the energy efficiency of food production and distribution. This can be accomplished by shifting from energy-intensive industrial agricultural techniques to less intensive methods (e.g., pasture-raised livestock, drip irrigation, non-synthetic fertilizers, no-till crop management, etc.), using more efficient machinery and equipment, reducing food processing and packaging, promoting decentralization of food production and improving the efficiency of food transportation. Farms can also generate their own clean electricity. While houses, barns and other buildings provide ample roof space for the installation of solar panels, farms with large swaths of land in windy areas are ideal sites for wind turbines. By leasing property for wind power production, these farms can earn an additional source of revenue while continuing to grow crops on surrounding land.

Despite the challenges posed by the energy-intensive nature of agriculture, the prudent use of resources and judicious application of technology has the capacity to significantly improve the long-term sustainability of food production.

2.2.5 Water Worries: World Comments

World´s Water Day

For 22 years, the UN drew attention to the strategic importance of fresh water and advocated the sustainable management of this precious resource. Furthermore, this is the ninth year of the International Decade for Action, “Water for Life” (2005–2015). The objective is to “promote integrated actions in relation to the use and conservation of water, reducing the scourge of millions of people in the world who live without access to safe drinking water”.

Water is a vital substance, but in recent decades changes have been observed as a result of humanity’s actions. The scarcity of clean water is already one of the great challenges of the 21st century. Considering that over 1.4 billion people (24 % of the world population) lack access to clean water, the issue has definitely become one of the global environmental topics. The amount of available fresh water is only 1 % of the total water on the planet.

But what is the relationship between water and energy? This year, World Water Day addresses as a main focus “Water and Energy”. The 2014 selection was because water and energy are closely interconnected and interdependent, as hydro, nuclear and thermal generation all need water.

According to the International Energy Agency, for example, a nominal increase of 5 % of road transport in the world by 2030 could increase the demand for water by up to 20 % of the resource used in agriculture due to the use of biofuels. 8 % of the energy generated on the planet is used to pump, treat and carry water for people’s consumption. In addition, water resources are used for geothermal power generation, which is an alternative for energy in countries with water shortages.

Actions like that of the State of Ceará, located in Northeastern Brazil, can make a difference. The activities of the Ziatech Company, through its projects and renewable energy in the area of project development, energy efficiency, planning, training and other services in regards to wind energy, contribute 37 trillion liters of water per year to the economy i.e., the equivalent of almost twice the volume of Brazilian´s Hydropower Itaipú water consumption.

The demand for water and energy will grow dramatically in the future. Renewable energy sources will play a vital role in supplying this demand. The consultant, José Galizia Tundisi, an expert in the working of the mechanisms of lakes, rivers, dams and integrated water resources management, says that "poor water impoverishes local populations in certain areas, in addition to interfering with the regional economy and destroying healthy alternatives for sustainable development". This leads to the following analogous reasoning: poor quality fossil energy not only impoverishes local populations and certain regions, but overall, too, interferes with the regional and global economy and destroys healthy alternatives for sustainable development.

When the United Nations (UN) defines the theme "Water and Energy" it is because they know that water and energy are closely interrelated and interdependent. Generation and transmission of energy demand water resources, especially for hydro, nuclear and thermal energy sources.

In 2014, while working with a concentrated focus on the "water-energy nexus", the UN also wants to draw attention to inequalities, especially for the billions of people of the southern region of the planet living in slums and impoverished rural areas who survive without access to safe drinking water, adequate sanitation, sufficient food or energy services. It also aims to facilitate the development of public policies that lead the way to energy security and sustainable water use within the context of a green economy.

Objectives of the World´s Water Day in 2014

- Raise awareness of the interrelationship between water and energy;
- Contribute to a political dialogue that focuses on the broad range of issues related to the water-energy nexus;
- Demonstrate, through case studies, to decision makers in the energy sector and the field of water that integrated approaches and solutions to the problems of water and energy achieve

higher economic and social impact;

- Identify policy and capacity development issues in which the UN system of Nations, particularly UN - Water and UN - Energy, can make significant contributions;
- Identify key interested parties in the water-energy link and involve them actively;
- Contribute significantly to the post-2015 discussions in relation to the water-energy nexus.

2.3 Energy Overview

In its ordinary definition, energy is the capability to produce work. However, the use of the concept is quite varied and sometimes wrong depending on its objective [73].

There are several ways to classify energy resources. One possible classification distinguishes between primary, secondary and end-use energy, with primary energy being in raw form, secondary energy represented by a stage between the primary form and the end use form, where this latter represents the primary energy already converted into energy that can be used for capitalized equipment that can produce work or heat [74]. It is possible to consider primary energy as a raw material, secondary energy as technology and end-use energy as the final product.

Therefore, primary energy is considered the input for production, because the various energy sources indicate the potential energy of a given system. Another way of defining energy resources is by form of use, i.e., commercial vs. non-commercial. One can also qualify further by generation of resources; renewable resources energy vs. non-renewable resources energy; by availability of resource use; limited resources energy vs. unlimited or not restricted resources energy; by technological state; conventional vs. non-conventional energy. It is understood by conventional sources energy those whose technology is fully developed with costs considered acceptable by current consumption standards and unconventional ones whose technology is already demonstrated, but still presents problems of acceptance in modern society, due for economic reasons, or because they are not according with the accepted standards of consumption [75].

Several of these classifications used here depend on what parameters we want to analyze on each occasion. Ultimately, the distinction will be the technological aspects and the limitations resources, which are the most relevant discussions of the work.

Thus we distinguished three generic types of primary energy supply:

1. Conventional Constrained Resources Energy;
2. Conventional Renewable Constrained Resources Energy;
3. Unconventional Unlimited Resources Energy.

Most energy currently in use comes from the first group, such as oil and oil products, charcoal, steam coal and metallurgical coal. In the second group are hydropower, wood and biomass products. And finally, from the third group come the vast majority of non-conventional energy sources such as solar, wind and nuclear (uranium).

2.3.1 Hydropower Energy

The generation of electricity from hydropower plants is potentially limited. It is not possible to build hydropower plants just anywhere, and with the growing demand for correct geographical positioning it is critical to decide where and how.

Hydropower is characterized as a renewable source; however, the utility of this source is limited by multiple uses of its raw material, water, and the geographic location of rivers. This characterizes hydropower as renewable but potentially limited. When its limit is reached, i.e., when the geographic availabilities are depleted, a replacement is needed to maintain the level of growth [76].

Hydropower is a major water user, relying on water passing through turbines to generate electricity. Water is also consumed via seepage and evaporation from the reservoir created for hydropower facilities. Factors determining the amount consumed –climate, reservoir design and allocations to other uses – are highly site-specific and variable. By one estimate, hydropower facilities in the United States consume 68 000 l/MWh on average, with a wide range that depends on the facility [77].

Another point worth mentioning is that although it is considered by many experts as a renewable energy form, hydropower can lead to economic harm. It should be borne in mind that water is a resource with great economic potential, serving for human consumption, food production and other demands. From the moment that the dam water is used for hydroelectric generation, it is also being channeled out of the economic supply chain which also depends on this same water.

In general, the functional objective that is used to control hydroelectric systems is based on minimizing the cost of the megawatt and minimizing the deficit probability. However, this control parameter may be forcing restrictions in the growth of the Global Economic Function, where the allocation of water for this purpose prevents the economy from growing in other more profitable segments.

2.3.2 About Energy System Choice

The primary sources of energy, and the technology needed to use them, constitute the energy system. The choice of energy systems is a fairly complex problem and naturally full of social, economic and political aspects. The various energy options available and the constant evolution of technology usage, makes any attempt to explain the advantages and disadvantages of each option without proper study a complicated task. Planning requires thorough long term study. Depending on the energy composition of each economy, the energy market exhibits different behaviors. "Conventional energy sources, with their limited supply, create a highly capitalized commodity and a relatively greedy and impatient market. However, unconventional sources, having unlimited supply, and because they are not scarce commodities which might become exhausted, are not subject to market forces or foreign policy " [78, 79].

2.3.3 About Brazilian´s Water and Energy Context

Brazil is the example shown below of a water-energy scenario where the final solution should be reviewed.

Before the 1970's, Brazil has used oil to supply its electrical energy matrix. After the oil crisis of 1973 and the second peak of the oil crisis in 1979, Brazil had to urgently restructure its production of electrical energy as oil costs were causing the economy to collapse.

Brazil is the country that has the most water in its rivers, lakes and underground aquifers, corresponding to 12% of the world total. Among Brazilian regions, the disparity in the distribution of fresh water sources is enormous: while the Northern Region accounts for 72% of the total water in the country, the Northeast has only 3%. In the semi-arid Northeast, as it is known, in addition to the present critical water scarcity, rainfall is very irregular, which leads to long periods of drought. In light of this scenario it is evident the importance of planning the use of water resources in these regions.

Due to good aquifer availability, after the oil crisis it was decided to design a hydroelectric matrix as a basis to meet national demands for electricity. This solution on this occasion brought huge reduction in costs to the national economy, and reasonable security in meeting demand.

But, with the passage of time, due to population growth and increased demand for energy and water, the Brazilian electric power system, which relies on around 90% of hydroelectric generation, began to present problems. The growth in demand, along with the seasonality of rainfall, increased the deficit of electricity. In 2001 Brazilians suffered the risk of an energy disruption, and emergency power saving policies had to be imposed on the population in the form of large rate increases to contain the risk of a power outage. This brought negative economic impacts and created fear in investors, employers and the population in general. Rainfall is dependent on seasonality and climate, and because the Brazilian energy matrix depends on hydropower, the risk of collapse is still imminent.

A definitive solution is possible involving long term planning to change Brazil‘s electrical energy matrix to rely on renewable and low water solutions that will provide better supply to meet demand for economic growth.

2.4 General Energy-Water Context

Energy and water are valuable resources that sustain human prosperity and are largely interdependent. Water is ubiquitous in energy production: in power generation, extraction, transport and processing of fossil fuels; and, increasingly, in irrigation for the cultivation of raw materials used for the production of biofuels. Similarly, energy is vital to the water supply needed to create power that collects, transports, distributes and cares for its systems [38, 67]. Each faces increasing demands and restrictions in many areas as a result of economic and population growth and climate change, which amplifies the mutual vulnerability of energy and water. For the energy sector, restrictions on water can challenge the reliability of existing operations and feasibility of proposed projects, imposing additional costs for necessary adaptation measures [39, 52, 53].

The other half of the water energy nexus concerns the energy needs for the supply and treatment of water. Electricity is needed to power pumps that facilitate (from ground and surface sources) transmission, distribution and collection of water [54]. The amount required depends on the distance (or depth) of the water source. Fresh, brackish, saline and waste - - water treatment, which convert water from various processes into water fit for a specific use, require electricity and sometimes heat. Desalination, a process that removes salt from water, is the most intensive and expensive energy option for the treatment of water, and is used where alternatives are very limited. Other energy needs associated with water occur at the end use point, often in families, mainly for heating water and doing laundry [40, 52]. Looking ahead, several trends point to growing demands on energy in the water sector:

- Increased water demand as a result of population growth and better living standards;
- Fresh water reserves scarce in close proximity to population centers, due to climate change. This means that the water must be carried long distances or pumped from great depths to undergo additional treatment;
- Stricter standards for water treatment;
- A general change in irrigation practices or surface flooding (relying on gravity) methods of pumping, which are more efficient in terms of water, but require energy for operation.

Water is needed to produce almost all forms of energy. For primary fuels, water is used in the extraction of resources, irrigation of raw materials for biofuels, refining, fuel processing and transportation. In power generation, water provides cooling and other needs related to the process in thermal power plants; hydroelectric energy is basic to the production of electricity. These uses may in some cases entail a significant amount of water.

2.5 About Energy-Water Nexus

It takes a significant amount of water to create energy. Power plants make steam from water – whether powered by coal, oil, natural gas and nuclear power – and it is also necessary to generate hydropower. Water is also used in large quantities during the extraction of fuel refining and production. It takes a significant amount of energy to extract, move and treat water for drinking and irrigation. Energy is necessary for the collection, treatment and disposal of waste water. Energy is also consumed when water is used by households and industry, especially through heating and cooling.

Water policy and energy planning and management must be integrated to encourage conservation, encourage innovation and ensure sustainable use of water and energy. End users, such as businesses and households, also have an important role to play. By reducing the amount of water they consume, end users will not only save water, but energy as well. This, in turn, can save money on utility bills.

It is necessary to examine both withdrawal and consumption of fresh water with special attention. Withdrawal is the total amount of water a plant takes from a source such as a river, lake or aquifer, some of which is returned. The intake is the amount lost by evaporation during the cooling process. Withdrawal is important for several reasons. Water intake systems can trap fish and other aquatic wildlife. For cooling water removal, water not consumed returns to the environment at a higher temperature, potentially harming fish and other wildlife. And when plants get into the cooling ground water, it can deplete aquifers critical to meeting different needs. Consumption is also important because it reduces the amount of water available for other uses, including the ecosystems that sustain them.

While some analysis focuses on the effects of water use by plants today, it must be considered how conditions may change in the future. In the short term, the choices for what kind of plants we build can contribute to stressing the fresh-water supply (by assigning too much of the available supply of water for generation use) and can affect water quality (by increasing water temperature to levels that impair local ecosystems, for example). Over a longer period of time, these choices can fuel changes in the quality of life, which in turn may also affect water quantity (through droughts and other extreme weather events) and quality (by increasing the temperature of lakes, streams and rivers). Population growth and increasing demand for water also promise to aggravate the level of water stress in many regions of the country, already under stress because of plant use and other uses.

The water supply is said to be stressed in river basins where the demand for water destined for energy plants, agriculture and municipalities, for example, exceeds a critical threshold from local surface and groundwater sources. Changes in water quality can also be noted as, for example, when water users raise the temperature of exhaust pollutants.

Global water withdrawals for energy production in 2010 were estimated at 583 billion cubic meters (bcm), or some 15% of the world's total water withdrawals. Of that, water consumption – the volume withdrawn but not returned to its source– was 66 bcm. In the New Policies Scenario, withdrawals increase by about 20% between 2010 and 2035, but consumption rises by a more dramatic 85%. These trends are driven by a shift towards higher efficiency power plants with more advanced cooling systems (that reduce withdrawals but increase consumption per unit of electricity produced) and by expanding biofuels production. The water requirements for fossil fuel-based and nuclear power plants – the largest users of water in the energy sector – can be reduced significantly with advanced cooling systems, although this entails higher capital costs and reduces plant efficiency. Future water needs for biofuels depend largely on whether feedstock crops come from irrigated or rain-fed lands and the extent to which advanced biofuels – whose feedstock crops tend to be less water-intensive – penetrate markets. Water requirements for fossil fuel production are comparably lower, though potential impacts on water quality are an important concern [41–43].

The conflict between the demands for energy and water for other uses point to the importance of accurate, updated information about the water demand of any energy plant.

Avoiding water conflicts requires energy plant operators to regularly supply accurate information about their water usage to better meet the needs of decision makers in the public and private sectors. However, better information is only the first critical step. Decision-makers must then combine that with sound analysis of water conflicts, working to control the thirst for electricity, especially in regions with water scarcity combined with a lack of reliable information [55–61]. Analysis provides some smart choices to put together a strong energy - water base. Here are some ways to do this:

- Developing new resources to meet electricity demand provides a major opportunity to reduce the risks of water conflicts for both plant operators and other users. Utilities and other power plant developers would be well advised to prioritize low-water or no water cooling options, particularly in regions of stress and projected high water usage;
- Retool existing factories. The owners and operators of existing plants having substantial impact on the supply or quality of water in water-scarce regions should consider adjusting the low-water cooling mark;
- Set strong guidelines for the use of water in the plant. Public officials should rely on good information about electricity demand to help owners of existing plants and make proposals to prevent conflicts with water and energy. Public utility commissions, which oversee utilities plans and proposals for specific plants, can encourage or require investments that reduce the adverse effects on water supply and quality, particularly in areas of current or projected water conflict;
- Involve interested parties. Mayors who control the water supply for the cities involved with sport fishing and commercial fishermen, water managers at all levels, and others, have an interest in avoiding water and energy conflicts. Full public access to information about water use by existing and proposed plants will allow these and other local interested parties to learn about the benefits of smart energy water options.

Avoiding water and power conflicts means having a long-term vision and mathematical modeling. Power plants are designed to last for decades and much of our existing infrastructure will continue operating for years. The precious fresh water resources of Brazil will face increasing stress from growing populations, climate change, and other trends over the coming decades. The typically high cost of adaptation of power plants means that decisions about the impact of water plants today should consider the risks to fresh water resources and energy reliability throughout their expected existence [62–65].

Decisions made today about which plants to build or to retire, and which energy or refrigeration technologies to deploy and develop are very important. Understanding how these choices affect water use and water scarcity will help ensure that the dependency of plants on water does not compromise their future, the plants themselves, or the energy that we rely on them to provide.

2.6 The Water and Energy Standoff

The choices on the future mix of plants used to generate electricity can ease the conflict between water and energy. Renewable energy technologies such as wind turbines and photovoltaic panels use little or no water and do not emit carbon pollution in electricity production.

Even fossil fuel technologies provide opportunities to reduce water demand while also addressing carbon emissions. Combined-cycle natural gas plants are use less water than coal plants, for example, emit fewer carbon emissions and, because of higher efficiency, produce less heat. New refrigeration technologies, such as hybrid dry cooling systems, can also reduce pressure on water systems.

Much is at stake. If energy companies have difficulty finding enough water to cool their power plants, blackouts can force them to buy electricity from other sources, which can raise utility bills. The increase in water temperature endangers fish and other aquatic species. The struggles between energy plants, cities and farms over limited water resources can be expensive, can force residents to reduce water consumption, and may damage the environment. To make water vs. energy choices wisely, information must be available regarding essential problems: how plants use water, where they get that water, and how it affects the use of water resources [41, 43, 46–48, 50–52].

2.7 The Art of Mathematical Modeling for Planning and Control

Optimal Solution requires Mathematical Modeling The Energy-Water Nexus is a theme that has attracted world-wide attention in various contexts like economy, ecology, environment, energy policies as well as involving small, medium and large industries. Although the attention and space that the topic received is significant, many distortions of information, conflicts of interest and even contributions of many to the problem are not resolved there. One way to consolidate opinion and strengthen arguments is through mathematical modeling of these situations. A mathematical tool has the power, in a few lines of information, to clarify arguments irrefutably, as a basis for informed decisions to be made. That is what is lacking to support government polices and support all who have viewed, even empirically, the importance of the topic and to give it the attention that it deserves.

2.8 The Economic Agents (Players) in The Game (especially in Brazil)

There are many players in this game, namely:

- The Citizens (people in general; final consumers);
- Public Institutions
  - Government (central)
    1. Ministries (Secretaries);
    2. Regulatory Agencies (ANEEL, ANA, etc.);
    3. Research and Development Centers (CEPEL, EPE, etc.).
  - City Hall / State Government
    1. Public Lighting;
    2. Water Supply;
    2. Transport.
  - Public Schools and Hospitals
  - Public or Mixed Economy Utilities (Energy and Water)
  - Oil and Fuels (Petrobrás)

- Private Institutions
  - Electric Energy Industries;
  - Fuels;
  - Industry in General;
  - Transport;
  - Agriculture (Agribusiness, Biomass, Biofuels, etc.).
- Non Governmental Organizations
  - Green Peace;
  - UN;
  - IEA, etc.

This is just a brief list of the players involved. In this thesis, the complete relationship between all those economic agents is not going to be covered, nor will there be specific recommendations for any of the sectors. This is not the focus of this thesis. One overall recommendation for the community as a whole is that the energy-water nexus issues should be treated as a unique problem, and that mathematical models, followed by the corresponding data analysis, should be used. In the next two chapters two mathematical models are proposed, and one reaches several conclusions from them.

2.9 Thesis Approach

The Energy-Water Nexus is a real problem with a complex solution. It's necessary to analyze it in a general context (Macro-Solution Optimization), that will help long term planning (05–10 years) and the supply chain situation (Local-Solution Optimization) that will help short term planning (01–05 years).

In specific situations some are starting to use mathematical modeling on the problem of water use in an energy context [24–26, 29, 92].

The next two chapters will detail two mathematical models to analyze the Energy-Water Nexus Issues: An Input-Output Dynamical Model (short term planning) and An Optimal Control Model (long term planning).

CHAPTER 3

Energy-Water Nexus: An Input-Output Dynamical Model

3.1 Introduction

Energy and water are of utmost importance for any country's economy and way of life. Understanding the intricate relationship between energy and water and developing technologies to keep that relationship balanced is an important key to a sustainable and secure future for any country. To generate energy, massive quantities of water are consumed, and to deliver clean water, massive quantities of energy are needed. We are analyzing the links between energy and water — called the energy-water nexus, or water-energy nexus — in a general context. For that study a dynamic input-output Leontief model is proposed. We concluded that a fully integrated management of energy and water, including support to analyze the impact of technological development, supply policy, and production planning, can be achieved through this approach.

There are trade-offs between energy and water. Large scale power plants — nuclear, coal, biomass and, of course, hydroelectric — use lots of water. Conversely, making drinkable, potable water, and piping it into big cities, typically involving large distances certainly requires plenty of energy.

Energy and water are strongly tied and certainly dependent on each other, with each affecting the other's availability. Water is necessary for energy development and generation, and energy is needed to supply, use, and treat drinking water and waste water. Both, energy and water are crucial to our health, quality of life, and economic growth, and demand for both these resources continues to rise.

Water and energy are the two most fundamental resources of modern civilization. People die, and one cannot grow food, if water is not available. Without energy, one cannot run computers or power homes, schools, offices, farms, and industrial plants. As the world's population grows in number and affluence, the demands for both resources are increasing at a faster pace than ever.

The earth holds about eight million cubic miles of fresh water — tens of thousands of times more than humanity‘s annual consumption. Only about 2.5% of the world's water is fresh water. Unfortunately, less than 1% is reachable via surface sources and aquifers, and the rest is trapped in underground reservoirs and in permanent ice and snow cover. Also, the available water is frequently not clean or not located close to population centers [43].

The reality that one of these precious resources, energy or water, might soon cripple the use of the other has been under-appreciated. To generate energy, massive quantities of water are consumed, and to deliver clean water, massive quantities of energy are consumed. Desalination, a process that removes salt from water, is the most energy-intensive and expensive option for treating water and is used where alternatives are very limited. Other energy needs associated with water occur at the point of end-use, often in households, primarily for heating water, cooling water, washing clothes, and pumping water. Many people are concerned about the perils of peak oil — running out of cheap oil. A few are expressing themselves about peak water. But almost no one is addressing the conflict between the two: water restrictions are impeding solutions for producing more energy, and energy problems, particularly rising prices, are imposing restrictions on the efforts to supply more clean water [41, 43, 48].

Physical constraints on the availability of water for energy sector use encompass both quantity and quality issues: there may not be enough of it or that which is available may be of poor quality. These restrictions may be natural or may arise from regulation of water use. One cannot build more power plants without taking into account that they negatively impact freshwater supplies. And one cannot build more water delivery and cleaning facilities without increasing energy demand. Solving the conflict requires new comprehensive policies that treat energy and water solutions as if they were just one problem, and innovative technologies that help to boost one resource without draining the other. One needs an analytical tool, a mathematical tool, to treat this problem. A mathematical dynamic model is proposed here.

Water and energy are intrinsically interconnected. Policy reforms in both sectors, however, do not appear to acknowledge the connections or consider their broader implications. This is clearly unhelpful, particularly as policy makers attempt to develop effective responses to water and energy issues, underpinned by prevailing drought conditions and menacing climate change, and more and more increasing demand for both resources.

Energy production necessitates an abundant, reliable, and predictable source of water, a resource that is already in short supply in many parts of the world.

The time has come to consider both issues as one. Instead of water planners taking for granted they will have all the energy they need and energy planners taking for granted they will have all the water they need, one must get them around the same table to develop models and make decisions. A restructuring of the institutional arrangements is in order. And we will need an analytical tool. This is the underlying idea of this proposed analytical model.

We investigate here the links between water and electricity — termed the water-energy nexus, or energy-water nexus — in the general context. In these circumstances a dynamic input-output Leontief model is proposed.

3.1.1 The Supply Chain Model

Supply chain management is a key tool in business administration. The input-output matrix, which is part of an underlying mathematical model, recapitulates the coordination of supply and demand with the various sectors of the economy; the gross purchase or sales of physical products among the various sectors of the economy. It also describes the technology of production [9, 85, 86].

Two important economic variables are supply, $\boldsymbol{s}$, and demand, $\boldsymbol{d}$. It is assumed that if demand grows, then supply will grow, or should grow, to match it. And vice-versa. In other words, supply should keep pace with demand. This depends on the phenomenon of feedback.

Since demand always varies, there will always be a dynamic equilibrium. The system is, in general, always moving. It is the dynamics of this movement that one wants to study and control. It is the essence of supply chain management. In the case of energy-water interplay this is crucial, since these two variables are closely intertwined.

3.2 The Input-Output (Leontief) Model

The input-output model started with the seminal work of Leontief, a nobel laureate who produced the input-output matrix of the American economy [26, 92]. References include [87–89]. A former reference is [80], which can be seen as a predecessor of this sort of ideas.

3.2.1 The Basic Identities of an Input-Output Matrix

The input-output matrix summarizes the matching of supply and demand amongst the various sectors of the economy; the gross purchase or sales of physical products amongst the economy various sectors. It also describes the technology of production. The following equations define relations between producers, and also the various sectors supplies, in the production environment. The mathematical formulations are collected in the sequel.

$$x_i = \sum x_{ij} + s_i \tag{3.1}$$

where,

- x_i is the ith sector gross production;
- x_{ij} represents the sales of sector i to sector j;
- s_i represents the ith sector finished products stock;

One has:

$$\boldsymbol{T} = x_{ij} \text{ is the inter-sector flux transition matrix involving}$$
$$\text{intermediate products;}$$

For each sector i, there exists a vector $T_i = (x_{i1}, x_{i2}, x_{i3}, ..., x_{ij})^T$

- $\boldsymbol{s} = (s_1, s_2, \ldots, s_i)^T$ is the ith sector finished product stock vector;
- $\boldsymbol{x} = (x_i)$ is the production vector.

Finally, the generic expression will be given as follows:

$$\boldsymbol{x} = T_1 + T_2 + T_3 + ... + \boldsymbol{s} \tag{3.2}$$

Assumptions of fixed proportion of factors input It is important to notice that equation (3.2) requires the inputs of any product vary proportionally to the output volume of the finished product, i.e., if the level of the finished product demand changes, all the inputs in that formula should be altered in the same proportion.

3.2.2 The Energy-Water Matrix

Only the interconnections between energy and water will be treated here. Three types of water are considered:

- $W_1 \to$ water that has a final use in the economy. This includes water for human and animal consumption (drinkable, potable water), water for irrigation (excluding energy crops, like sugar cane for the production of alcohol, for instance), water for industrial processes (excluding the water necessary for the production of any fuel, or energy product).
- $W_2 \to$ water necessary for the production of energy (or fuel), excluding hydroelectric water. It includes water necessary for cooling power plants (thermal turbines, nuclear generators, etc.), production of biomass for energy crops (irrigation and industrial processes, like in the production of alcohol from sugar cane, for example), and the production of fuels.
- $W_3 \to$ hydroelectric water. It is the water (freshwater; not sea water) that is stored and used in dams to generate electricity through water turbines ("water with a head"), or in "a fio d'água" (trickle) turbines.

Clearly the total water stock depletion rate will be:

$$W = W_1 + W_2 + W_3. \tag{3.3}$$

The total stock of water (natural deposit; natural reserves) will be denoted by D. One is not going here into the details of the geographical distribution and benefit distribution of this stock, nor the engineering and political problems involved in its handling. See [90] and [91] for some studies. Water for whom? Who are the agents? Who are the players in this game? What are the prices? This is not the focus or the scope of this work. The objective here is to provide a scientific and technical background that can support the actions that could lead to an adequate, optimal, set of solutions to the problems involving the energy-water nexus.

As for the energy (excluding hydroelectric energy), one can adopt the classification used in [24]: restricted and non-restricted:

- Restricted (E_R): includes fuels in general, and also biomass.
- Non-Restricted (E_{NR}): includes solar, aeolic, nuclear, sea wave, tidal energy, and perhaps geisers.

Clearly the total energy (excluding hydroelectric) stock depletion rate will be:

$$E = E_R + E_{NR}. \tag{3.4}$$

The total stock of energy (natural deposit; natural reserves) will be denoted by D_E. Again, as in the case of water, one is not going here to be concerned with the details of the geographical distribution and benefit distribution of this stock, nor the engineering and political problems involved in its handling. See [90] and [91] for some studies. Energy for whom? Who are the agents? Who are the players in this game? What are the prices? This is not the focus or the scope of this work. The objective here is to provide a scientific and technical background that can support the actions that could lead to an adequate, optimal, set of solutions to the problems involving the energy-water nexus.

As in [24] and [25], energy and water will be measured in the same units. One has then a balance equation:

$$E_N = E + W_3 - W_2 - W_1, \tag{3.5}$$

where E_N is the net energy (depletion rate) to be used for the production of non-energy goods, excluding non-energy water (W_1).

It should be kept in mind that E_R, E_{NR}, W_1, W_2, and W_3 are aggregates.

Before giving an illustrative example of an input-output matrix for the energy-water nexus, some comments are in order.

Some Important Concepts

One should bear in mind some basic features of a supply chain [9, 26, 85–89, 92]:

- The degree of interdependency between several industries.
- No firm is an isolated island. The production tends to be specialized, and the firms in a supply chain depend upon one another. Changes in one of them will gear a series of repercussions throughout the whole chain. As small as they could be, those changes will spread out from one firm to the other in the system in such a way that its accumulative effect in the chain can be significant; substantial. It is important that the agents, the decision makers, be able to evaluate its global effect, direct as well as indirect, of a change in a part of a system.
- As no firm is an island, no supply chain is an island. The world is all interconnected.
- In an input-output analysis (Wassily W. Leontief), the supply chain is decoposed into firms and the goods and services flux between the subsystems is registered in order to systematically indicate the relations between them. The input-output techniques are a

special type of general equilibrium analysis in economy. In this type of analysis production is described in terms of a set of linear equations.

- By decomposing a chain into "finer" units, the input-output techniques are capable of tracking, or identifying, the non-detected effects hidden in a more aggregated analysis; in a macro-analysis.
- The analytical apparatus is based upon an input-output matrix (consumable-product, or supply-product matrix). From it and the appropriate theoretical assumptions concerning its meaning, there exists several techniques which have been developed and used in order to study and analyze a series of economic problems. This methodology was transferred to the study of supply chains [9].
- Amongst the concepts and techniques one may mention:
  - the income multiplier;
  - the price effect;
  - the technical matrix triangularizazion;
  - the interrelationship analysis;
  - local, regional, national, and international impact analysis;
  - the strategic planning.

The Input-Output Matrix

Consider the input-output matrix shown in table 3.1. The numbers are merely illustrative.

Table 3.1: An energy-water input-output table.

	E_R	E_{NR}	W_3	W_2	W_1	End Use	Gross Production
E_R	30	60	40	20	50	300	500
E_{NR}	15	30	0	20	50	85	200
W_3	10	5	5	5	50	625	700
W_2	40	15	0	5	0	0	60
W_1	0	0	0	0	0	200	200
Labor Force	80	40	20	10	60	90	300

The structure and functioning of the matrix is straightforward. In the first line, from the total (gross) production of 500 units of "firm"/product E_R, 30 are used to produce E_R, 60 to produce E_{NR}, 40 to produce W_3, 20 to produce W_2, 50 to produce W_1, and 300 is for end use (supply to the market). The logic for the other lines is the same.

It is plausible to consider the consumers as a sixth “firm”, or “productive unit“. The labor force would be its product and the various final goods and services would be the inputs used in the “production” of work. Note however that to consume goods is not exactly the same thing as to produce work (via a technical coefficient). In fact, the consumers can change their spending habits, their consuming habits, without changing their ability to work. In other words, consumption is not physically related to “quantity of work” as the other input-output relations (although in the aggregate this is true, since without eating men can not work). One distinguishes, thus, in practice, the consumer sector (the “firm” consumers), from the other productive firms. The final consumption (end use, in table 3.1) is considered to be a more autonomous variable, independently of the work force supply. It is, as a matter of fact, a source of uncertainty, which has its locus in the demand. Consumers can change their habits without provoking ruptures in the basic input-output relations.

This matrix can be expressed in technical terms. To obtain it one should divide the various consumables of each firm by its gross production. The results are shown in table

Table 3.2: The energy-water input-output table expressed in technical terms.

	E_R	E_{NR}	W_3	W_2	W_1	End Use	Gross Production
E_R	0.060	0.120	0.080	0.040	0.100	0.600	500
E_{NR}	0.075	0.150	0.000	0.100	0.250	0.425	200
W_3	0.014	0.007	0.007	0.007	0.071	0.893	700
W_2	0.667	0.250	0.000	0.083	0.000	0.000	60
W_1	0.000	0.000	0.000	0.000	0.000	1.000	200
Labor Force	0.267	0.133	0.067	0.033	0.200	0.300	300

3.2.3 Technical Input-Output Matrix

The quantity a_{ij} is the technical coefficient indicating the quantity of each product i necessary to produce one unit of product j. It is defined by:

$$a_{ij} = \frac{x_{ij}}{x_j} \Rightarrow x_{ij} = a_{ij}x_j \tag{3.6}$$

One defines, then, $A = [a_{ij}]$, as the technical matrix.

Using equation (3.6) in equation 3.1 one obtains:

$$x_i = \sum_j a_{ij}x_j + s_i \tag{3.7}$$

Substituting equation (3.7) into equation (3.2) it will follow that:

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{s} \quad \therefore \; \boldsymbol{s} = \boldsymbol{x} - \boldsymbol{A}\boldsymbol{x} = \boldsymbol{I}\boldsymbol{x} - \boldsymbol{A}\boldsymbol{x} = (\boldsymbol{I} - \boldsymbol{A})\boldsymbol{x} \tag{3.8}$$

where $\boldsymbol{I}$ is the identity matrix.

Finally, for each sector i, the net sales will be equal to $x_i - x_{ii}$, and when

$$x_i - x_{ii} > 0 \quad \therefore \; x_i - a_{ii}x_i > 0 \Rightarrow a_{ii} < 1.$$

Therefore a sector can produce some net output only if a_{ii} is less than one.

One can establish straightforwardly the relations between variables in physical units and variables in money units. This is not going to be done here.

3.3 Demand, Supply, Feedback and the Supply Chain Dynamics

The idea exposed in [92] for studies in economics is adapted here for the study of the energy-water nexus.

3.3.1 Input-Output Dynamics

Two important economic variables are supply, $\boldsymbol{s}$, and demand, $\boldsymbol{d}$. It is assumed that if demand grows, then supply will grow, or should grow, to match it. And vice-versa. In other words, supply keeps tracking demand. There exists thus a feedback phenomenon.

The equilibrium point is defined by:

$$\boldsymbol{s}(t) = \boldsymbol{d}(t). \tag{3.9}$$

Since demand is always varying, there will be always a dynamic equilibrium. The system is, in general, always moving. It is the dynamics of this movement that one wants to study and control. It is the essence of a supply chain management. In the case of the energy-water interplay this is crucial, since these two variables are closely intertwined.

The variable which provokes the increasing or decreasing of production is the difference between demand and supply. As the population grows, the demand for water (which has no substitute) and energy will grow, and not always in a balanced fashion. There will be a “demand surplus” or a “supply surplus”. This is the result of the feedback mechanism, as shown in figure 3.1.

The purpose of production is to provide products to be consumed. Part of the consumption is internal, that is, a fraction of product i is used to fabricate product j. Part of this "consumption" fraction is used to provide a stock (specially for production expansion). The other production part is for external consumption. The feedback gain K should be chosen in such a way that in the equilibrium point, which should be stable, supply equals demand.

Continuous variables will be considered here, and a set of differential equations will be obtained. The same reasoning that led to equation 3.8 leads to:

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\frac{d\boldsymbol{x}(t)}{dt} + \boldsymbol{s}(t) \tag{3.10}$$

where

- $\boldsymbol{x}(t)$ is the system total production;
- $\boldsymbol{A}\boldsymbol{x}(t)$ is part of the total production which is used in the production process;
- $\boldsymbol{B}\frac{d\boldsymbol{x}(t)}{dt}$ is the part of the total production which is put in stock (energy and water need to be stocked);
- $\boldsymbol{s}(t)$ is the part of production available to satisfy the external demand (it is the supply).

The error, that is, the difference between demand and supply, is given by:

$$\boldsymbol{e}(t) = \boldsymbol{d}(t) - \boldsymbol{s}(t) \tag{3.11}$$

A proportional control law will be used here. One could use other control strategies, like PID (Proportional Integral Derivative), for instance [81–84], but for the purpose of exposition clarity, only the proportional control will be used. The control force, u, will be then proportional to the error, and will be given by:

$$\boldsymbol{u}(t) = K\boldsymbol{e}(t) = K\left[\boldsymbol{d}(t) - \boldsymbol{s}(t)\right] \tag{3.12}$$

where K is the proportional control law matrix. It should be noted that K can be diagonal or not. There are no restrictions in that sense. In the sequel only the proportional control is used.

automatic control systems terminology). Gain K_1 gives speed of response and less steady state error, the larger its value; the integral control eliminates the steady state error, and the derivative control damps abrupt error variations [81–84]. In the sequel only the proportional control is used.

The production time rate will be then given by:

$$\frac{d\boldsymbol{x}(t)}{dt} = \boldsymbol{u}(t) \tag{3.13}$$

In order not to overload the notation, the explicit dependence on time will be eliminated, as well as the vector notation (boldsymbol). One will have thus:

$$\begin{aligned} \frac{dx}{dt} &= u \\ x &= Ax + B\frac{dx}{dt} + s \\ u &= K(d - s) \end{aligned} \tag{3.14}$$

Matrix K should be chosen in such a way that, in the equilibrium:

- supply equals demand ($s_e = d_e$, if d_e is constant);
- the Leontief static model be obtained, i.e.,

$$s_e = (I - A)x_e. \tag{3.15}$$

The diagram in figure 3.1 illustrates equations (3.14).

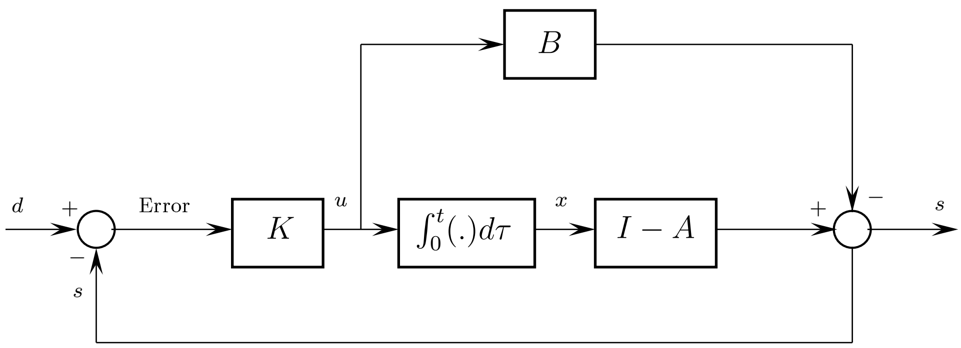

Figure 3.1: Block diagram of a supply chain dynamical model.

The closed loop control system will then be described by (see Appendix for the details):

$$\frac{dx}{dt} = -K(I - BK)^{-1}(I - A)x + K(I - BK)^{-1}d \tag{3.16}$$

$$s = (I - BK)^{-1}(I - A)x - (I - BK)^{-1}BKd \tag{3.17}$$

3.4 Simulations

The simulations were done as illustrations in order to show the advantages of an integrated (energy and water) control. All energy sources were aggregated, as well as all waters. The objective here is to demonstrate that a totally integrated parametric control of the system provides great flexibility, and so gives a good elbow room to optimize the system as a whole, for any type of optimality criteria.

The following values were used:

$$\boldsymbol{A}=\begin{bmatrix}0.30 & 0.15\\ 0.10 & 0.13\end{bmatrix};\ \boldsymbol{B}=\begin{bmatrix}0.020 & 0.010\\ 0.010 & 0.025\end{bmatrix};\ \boldsymbol{K}=\begin{bmatrix}0.30 & k_{12}\\ k_{21} & 1.25\end{bmatrix}$$

For the demand and supply series, increasing exponential functions with an added "noise" were used, being the energy demand showing a larger growth rate as compared to the water demand. The basic values were arbitrary. They are merely illustrative.

3.4.1 Stability Condition

One must satisfy the constraints of the system. The values of the real roots of the polynomial was placed on the worksheet user for the analyzed equation. One should make those roots always negative, in order to guarantee stability. It is better to preclude complex roots in order to avoid oscillations.

3.4.2 Demand Generator

In the "real world" the demand for the various products do not vary in a fixed proportion among them. So, for instance, d_1, the demand for product 1, could increase and d_2, the demand for product 2, decrease. This will vary depending upon several factors. In the case of energy and water, for example, one could have technological development, as well as production arrangements and public policies innovations, which in general will interfere (sometimes desirably) in this correlation. Thus, if the matrices K and B were chosen to be diagonal, one would be "tied" to the "physical" structure of the supply chain of the energy-water nexus, with very low possibility of optimizing the system as a whole. Both, K and B should be chosen full, that is, all their elements should be non null, so one will be able to accommodate all possible correlations between the demands for the two products, energy and water, whatever being theirs variances. This means that both, the production planning (policy) (K) and the stock control (B) should be established for the system energy-water as a whole, as a function of the vectorial demand. In the case of the energy-water nexus this is crucial.

For demand simulation in this model, an increasing function was used. There is no data base ready to represent precisely this function, but it is known that as the world population increase, the demand for energy and water grows roughly proportionally [12].

3.4.3 The $\boldsymbol{B}$ Matrix

As already said the control matrix B is the storage policy that the companies involved in the distribution network should develop in order to damp demand uncertainty or to create security mechanisms.

The problem with reducing the storage policy is the risk of go through periods of abrupt change in demand and the company be required to set up a matrix K with high values, destabilizing the productive system. In case of availability of energy, for example, the elimination of stocks of energy can cause an economic collapse. Due to economic risk, simulating an increase of the storage, despite being contrary to literature in general, should be considered, under penalty of economic collapse.

3.4.4 The $\boldsymbol{K}$ Matrix

The matrix K, as already mentioned, is responsible for the production planning. It was named force control. The values should typically be diagonal and off diagonal values act as muffling the interaction between customer and supplier. By acting on all its elements, one has a good flexibility in pole assignment.

3.4.5 The Objective Functional

The model just described leads the input-output matrix analysis in the direction of modeling a system of optimal control. A point worth mentioning is the flexibility of the model and the evaluation process of the resulting policy quality, because the structure was created with independence of the functional objective that one wishes to evaluate, and can be set to inform the result in real time. The sum of the quadratic errors (differences between demand and supply) was used as an objective functional, but any other criterion could be used.

3.4.6 Separated Policies for Energy and Water

In this case energy and water are managed separately. The results are shown in figures 3.2, 3.3 and 3.4.

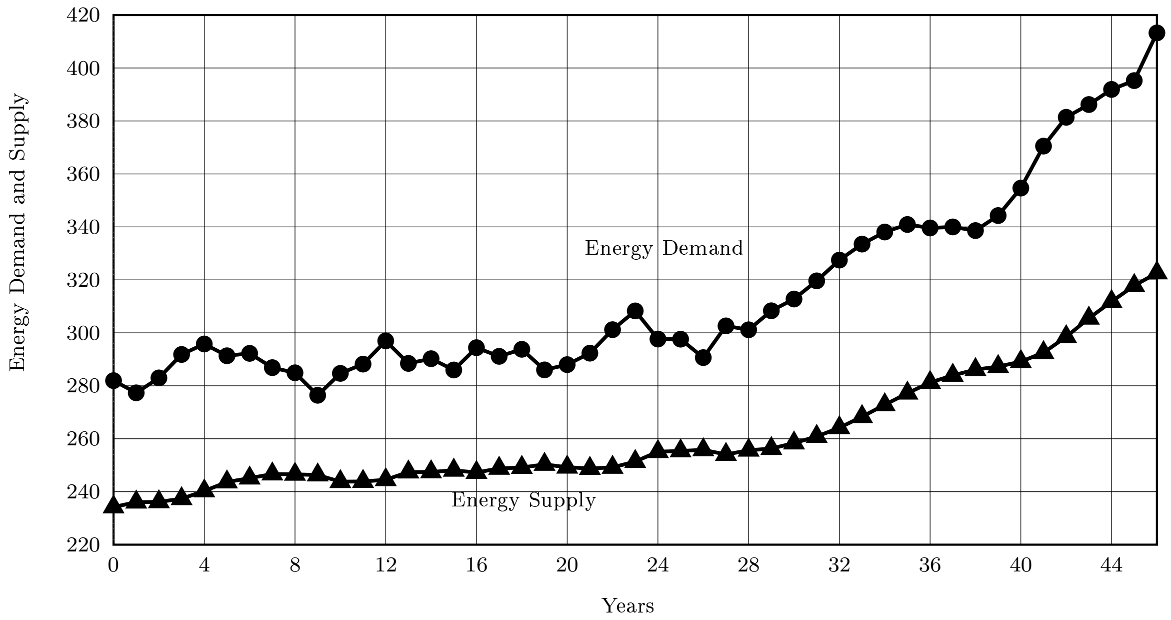

Figure 3.2: Energy demand and supply curves, when the K matrix off-diagonal elements are null, i.e., $k_{12} = 0$ and $k_{21} = 0$ (independent control of energy and water).

By changing the values of k_{11} and k_{22}, one could diminish the errors. This would have the effect of increasing the intensity of the control effort (cost). The eigenvalues λ_1 and λ_2 of matrix $-K(I - BK)^{-1}(I - A)$ are:

$$K = \begin{bmatrix} 0.30 & 0.00 \\ 0.00 & 1.25 \end{bmatrix} \quad \rightarrow \quad \begin{cases} \lambda_1 = -1.13 \\ \lambda_2 = -0.21 \end{cases}$$

The error can be seen from trajectories shown in the graphics in figures 3.2 and 3.3. The sum of squared errors was 316.

3.4.7 Integrated Policies for Energy and Water

In this case energy and water are managed in an integrated fashion. The results are shown in figures 3.5, 3.6 and 3.7.

Instead of increasing the intensity of the separable controls (k_{11} and k_{12}), one introduces non-zero elements in the off-diagonal of the matrix K, namely, $k_{12} \neq 0$ and $k_{21} \neq 0$. The eigenvalues λ_1 and λ_2 of matrix $-K(I - BK)^{-1}(I - A)$ are:

$$K = \begin{bmatrix} 0.30 & 0.50 \\ -0.80 & 1.25 \end{bmatrix} \quad \rightarrow \quad \begin{cases} \lambda_1 = -0.75 \\ \lambda_2 = -0.63 \end{cases}$$

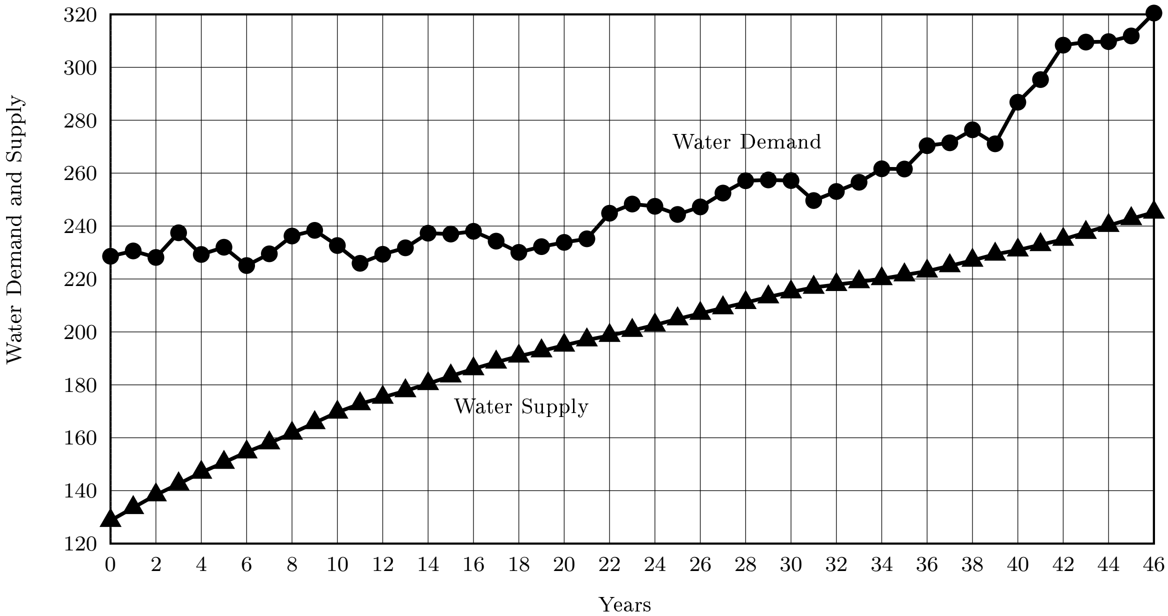

Figure 3.3: Water demand and supply curves, when the K matrix off-diagonal elements are null, i.e., $k_{12} = 0$ and $k_{21} = 0$ (independent control of energy and water).

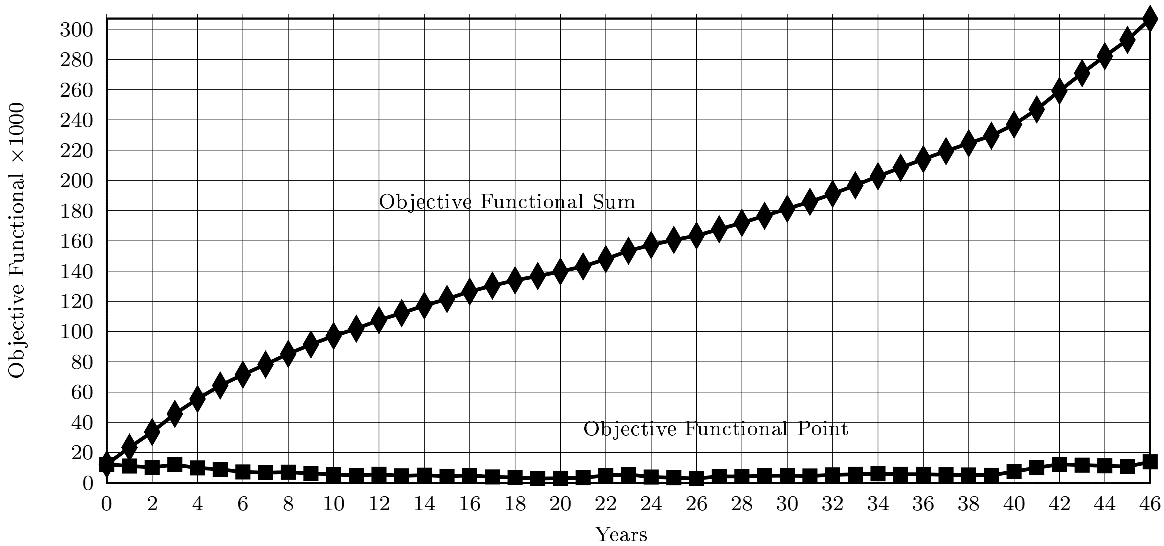

Figure 3.4: Energy demand and supply curves, when the K matrix off-diagonal elements are null (independent control of energy and water).

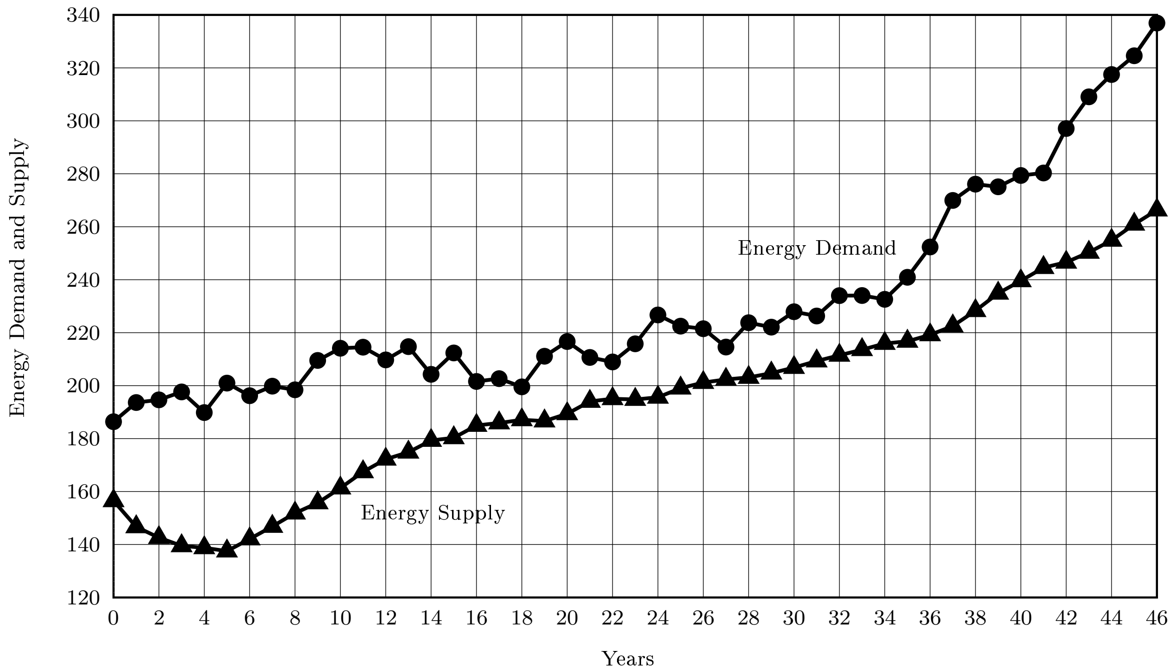

Figure 3.5: Energy demand and supply curves, when the K matrix off-diagonal elements are non-null, with values $k_{12} = 0.50$ and $k_{21} = -080$ (integrated control of energy and water).

The error can be seen from trajectories shown in the graphics in figures 3.5 and 3.6. It is visible that the error, in this case, is smaller than in the case of separated policies. The sum of squared errors was 126. The ratio was then 2.50.

Note that the values of the eigenvalues are more closer to each other when compared to the separated control case.

The results presented here depended, of course, upon the matrices A and B that were used. The objective of the simulation is to highlight the flexibility that one can achieve.

3.5 Three Time Scales for Control

It should be noticed that the control one is dealing with here, in the system defined by (3.16) and (3.17), is a parametric (closed loop) control. The demand vector d is in fact a perturbation.

There are three time scales for control. The manager will act in the elements of

- matrix A (technology development),
- matrix B (stock policy), and
- matrix K (production planning).

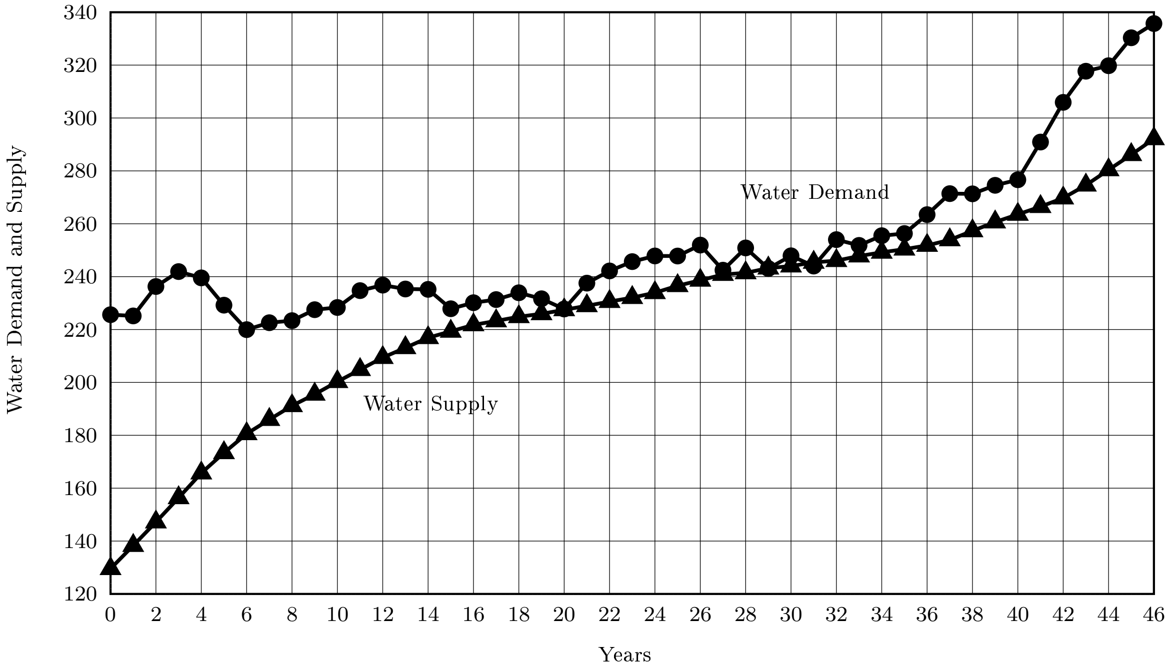

Figure 3.6: Water demand and supply curves, when the K matrix off-diagonal elements are non-null, with values $k_{12} = 0.50$ and $k_{21} = -080$ (integrated control of energy and water).

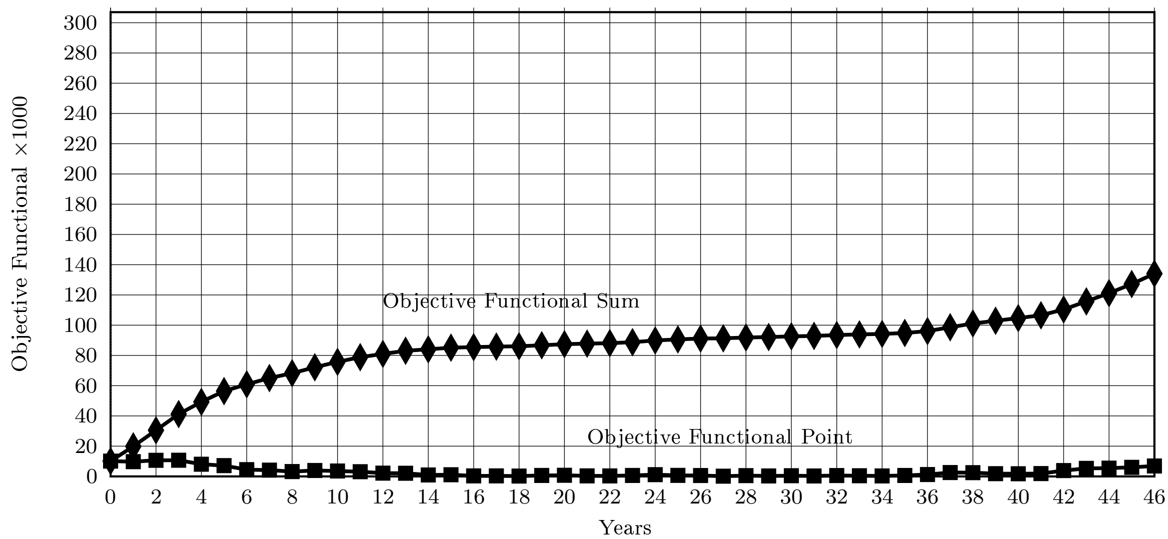

Figure 3.7: Objective functional curves, when the K matrix off-diagonal elements are not null (integrated control of energy and water).

In the simulations presented here, the optimality criterion was the quadratic error (to be minimized), and the A and B matrix were fixed. Only the elements of matrix K were used as parametric controls.

A more general objective functional can be used, including, for instance, the control cost.

One can use probabilistic algorithms in order to compute the parametric controls.

the economy, as well as the national income accounts, and a general equilibrium analysis. It is a natural setup for these studies, and thus a feasible tool for proposing national development policies. One could include, for instance, food and transport. four state variables. This model could also be implemented in a spreadsheet.

3.6 Conclusions

Earth and the communities that live on it are part of a very large system. By approaching these massive problems from an integrated standpoint, we begin to solve problems in a more systematic way.

The energy-water nexus is gaining popularity and acceptance with diverse interested parties around the world and it is becoming clearer that we cannot plan for the planet's future if we do not consider energy and water as a whole.

There is a strong connection, or nexus, between energy and water. It takes a substantial amount of water to produce energy. Water is used to cool steam electric power plants — fueled by coal, oil, natural gas, biomass, and nuclear power — and is required, of course, to generate hydroelectric power. Water is also used in large amounts during fuel extraction, refining and production.

It takes a significant amount of energy to capture, move and treat water for drinking and irrigation. It is also used in the collection, treatment and disposal of waste water. Energy is also consumed when water is used by households and industry, especially through heating, cooling, washing, and chemical processes.

Energy and water policy, planning and management, including operation, must be coordinated to encourage conservation, motivate technological innovation and guarantee sustainable use of water and energy.

It takes a substantial quantity of water to produce energy, and a significant amount of energy to extract, move, and treat water. One may not realize it, but when one uses energy, one is also indirectly using a lot of water!

Energy and water are fundamentally intertwined, but the linkage of these two vital ingredients also greatly impacts the food and transportation sectors. To an increasing extent one hears about the energy-water-food-transport nexus and how one is going to manage growing populations and the demands on all four resources. The model presented here is readily applicable in this case too. As a matter of fact, we can include as many sectors as we want. The input-output matrix can be made more and more diverse. The only limitation is the ability to obtain data. A large research grant is needed here.

Plus, when we take into account the global impacts from climate change, we are facing some big challenges — but also some opportunities. Increasing the use of clean and sustainable energy sources like wind, solar, sea wave, and tidal power is good policy; a crucial step towards reducing energy-related water use.

CHAPTER 4

Energy-Water Nexus: An Optimal Control Model

4.1 Introduction

An energy-water nexus mathematical model is proposed, formulated in terms of an optimal control problem representing an evolving economy; an optimal economic growth model. It is written as a maximization of a time-driven social welfare function, subject to constraints defined by income and investment participants, production technologies, the dynamics of the consumption of reserves, the energy balance and the labor force balance. The policy instruments are the investments in each sector, the consumption rate for the energy resources, and the water usage rate. The model is treated via the Pontryagin maximum principle. The results obtained from the model are useful in the understanding of the sector as a whole, and as a support in establishing integrated policies in the context of the energy-water nexus.

One may say that economics is the study of the allocation of scarce resources between competing uses. Scarcity is a characteristic of exhaustible resources.

Most of the world's energy resources known today are finite i.e., exhaustible, or limited. In the global energy matrix, oil comes first, followed by coal and natural gas. Facts and figures mentioned here can be found in [10] and [11]. These resources together account for approximately 80% of the world's energy supply. Coal is the resource used most to generate electricity. It accounts for generating 41% of the world's electricity supply. The United States and China are examples of countries that are highly dependent on this resource. In Brazil, water is the resource most used to generate electricity, followed by biomass [25, 37]. These resources, although renewable, are limited.

What oil, coal, natural gas, water and biomass have in common is that they are conventional. Conventional resources represent stored energy; they are found in specific and unchangeable locations and offer a limited supply. Their scarcity and high demand create a commodity for capitalization and an avid and impatient market, i.e., these resources are highly marketable in the international market and thus subject to price variations.

The issue is that these natural resources are allocated both for producing energy and for producing non-energy goods. Therefore, they are used as an input for production and as an input to produce a different kind of input, namely power.

Energy and water are at the heart of any country's economy and way of life. National defense, food production, human health, manufacturing, recreation, tourism, and the daily functioning of households all rely on a clean and affordable supply of both of them.

It is known that the production and consumption of energy and water are closely intertwined [1, 5–8, 13–15, 17, 18, 40, 52, 67]. They are diversified. Energy includes electric energy, and fuels like gasoline, diesel, naphtha, kerosene, alcohol, fuel oil, uranium, and the like. Water includes drinkable water (potable), water for irrigation, water for cooling, water for industrial processes, and so on. The end users of both, energy and water are many. There are also several producers of both.

Energy and water are, in their turn, intrinsically related to the production and consumption of food and transport.

Keeping electric power plants cool requires lots of water. Keeping water safe takes lots of energy. Eventually this may force a choice between the two.

Water is needed to generate energy. Energy is needed to deliver water. Each resource limits the other — and both may be running short. Is there a way out? What would be the rational way to deal with this problem?

In some countries, the two greatest users of fresh water are agriculture and power plants. Thermal power plants — those that consume coal, oil, natural gas or uranium — generate more than 90 percent of U.S. electricity, and they are water hogs [42, 43]. The sheer amount required to cool the plants impacts the available supply for everyone else. In other countries, like Brazil, for example, hydroelectric energy plays a major role, and it has been found that water to be used in agriculture, industrial processes, and human consumption, for example, has twice the economic value it has after it has been transformed into electricity in a hydroelectric power plant [24, 25].

At the same time, one uses a lot of energy to move and treat water, sometimes across vast distances. Health standards typically get stricter with time, too, so the amount of energy that needs to be spent per gallon will only increase.

A mathematical model is proposed here formulated in terms of an optimal control problem representing an evolving economy; an optimal economic growth model. It is written as a maximization of a time-driven social welfare function, subject to constraints defined by income and investment identities, production technologies, the dynamics of the consumption of reserves, the energy balance and the labor force balance. The policy instruments are the investments in each sector, the consumption rate for the energy resources, and the water usage rate. The model is treated via the Pontryagin maximum principle [4, 19, 21, 22, 29, 30, 68]. The results obtained from the model are useful in the understanding of the sector as a whole, and as a support in establishing integrated policies in the context of the energy-water nexus.

4.2 The Control Problem

The control problem is expressed by:

$$\underset{u}{Max}\ J = \int_{t_0}^{t_1} I(x,u,t)dt \tag{4.1}$$

subject to:

$$\frac{dx}{dt} = f(x,u,t) \tag{4.2}$$

$$x(t_0) \in \mathcal{X}_0 \tag{4.3}$$

$$x(t_1) \in \mathcal{X}_1 \tag{4.4}$$

$$u \in U \tag{4.5}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^p$, $t \in \mathbb{R}$, f satisfies a Lipschitz condition, I is Lebesgue integrable, $\mathcal{X}_0$ is a set which should contain the initial condition and $\mathcal{X}_1$ is a set which should contain the final condition of the state variable (vector). This is the Lagrange form control problem.

Another problem form is the Mayer form, when one has $J = F(x(t_1), t_1)$. The juxtaposition of these two forms gives the Bolza form, in which $J = \int_{t_0}^{t_1} I(x,u,t)dt + F(x(t_1), t_1)$. Function F is called the final function. These different forms permit an easier problem identification in different contexts, but from a mathematical viewpoint they are entirely equivalent [29].

Depending upon the forms of the several involved functions, one can characterize several types of problem, as, for example, the minimum time problem, the minimum energy problem, the maximum profit problem, and the economic growth problems. One may have other side constraints in the state and control variables.

4.3 The Pontryagin Maximum Principle

The Pontryagin Maximum Principle (PMP) is a tool to approach the control problem. It has an enormous theoretical importance (that is, the most practical possible!) as an epistemological tool, i.e., a large power to open up ways to reach knowledge. It is not just to make calculations; but also, and essentially, it is to establish concepts, grasping a subject of study and give meaning to it, and to establish scientific results. Its application spectrum is very large; it includes many fields of knowledge (engineering, economics, operations research, business administration, supply chain management, etc.). After all, it is a mathematical tool (not to be confused with statistical tools, which have nothing to do with the essence of science). References includes [4, 19, 21, 22, 30, 34, 66, 68].

The optimality necessary conditions obtained from the PMP are the ones which allows the establishment of optimal economic policies. There are also the transversality conditions, and the so-called turnpike theorems which refine the optimal solution, enabling the incorporation of more restrictions to the problem.

Consider the Lagrange problem. The proof of the results exposed here can be found in [66] and the other aforementioned references. They are the necessary conditions for the solution of the control problem; the so-called necessary optimality conditions.

The systematic is the following:

1. One writes the system's Hamiltonian, which is given by:

$$H(x, u, y, t) = I(x, u, t) + y^T f(x, u, t) \tag{4.6}$$

   where y is the costate variable, also called the Pontryagin multiplier (equivalet to the Lagrange multiplier in the static optimization case). Note that $y(t) \in \mathbb{R}^n$.

2. One maximizes the Hamiltonian by a choice of u, solving the problem as being one of static optimization. A nonlinear programming problem, to be precise. If this problem has an interior solution (interior to the set U of admissible, or feasible, control forces),

and I is differentiable with respect to u, its solution will be obtained from the necessary condition:

$$\frac{\partial H}{\partial u} = 0. \tag{4.7}$$

If there is no interior solution, one has to consider the constraints on the control forces, and apply the necessary conditions: the Karush-Kuhn-Tucker conditions [2, 3]. Depending on the problem, the analytical treatment can be complicated, even for low order systems (second or third order). In most cases the problem is analytically untractable, and it has to be solved through numerical methods. Once this problem is solved, one has then $u^* = u^*(x, y, t)$.

Along the optimal trajectory, thus, the Hamiltonian is constant.

3. One substitutes this value of u just found, into the $2n$ differential equations:

$$\frac{dx}{dt} = \frac{\partial H}{\partial y}. \tag{4.8}$$

$$\frac{dy}{dt} = -\frac{\partial H}{\partial x}. \tag{4.9}$$

In order to solve these $2n$ differential equations it is necessary to know the boundary conditions, which in the general case would be $x(t_0) = x_0$, $x(t_1) = x_1$, $y(t_1) = y_1$. Or there could be more complex conditions.

Once these differential equations are solved, all one has to do is to substitute their solutions into the expression for the optimal control u^*, and the control problem is thus solved.

4.4 The Model

The model follows the same rational as used in [25] and [24].

One starts with the notation. Let:

J = intertemporal welfare function.

δ = social rate of discount; the interest rate.

L = total labor force.

u = per capita instantaneous utility function.

c = per capita instantaneous consumption of non-energy goods.

t = time.

F = production function of non-energy goods (except the availability of water for productive ends).

E = total energy consumption rate of all energy resources in the aggregate (excluding water with hydraulic head).

W_1 = annual consumption or extraction rate of non-energy water that can produce non-energy goods (including human consumption, animal consumption, irrigation (excluding irrigation for energy crops, like sugar cane to produce fuel alcohol, for instance), products of industrial processes, etc.) in equivalent energy units, that is, W_1 is measured in the same unit as E.

W_2 = annual consumption or extraction rate of energy water, that is, water that is used for the production of any type of energy, except hydroelectric power.

W_3 = annual consumption or extraction rate of the water that is used for the production of hydroelectric power.

D_E = Energy stock (excluding water with hydraulic head).

D = Water stock (including the reservoirs of the hydropower utilities; water with hydraulic head, but also all other sources of water).

The subscripts used in the remaining variables have the following meanings:

a) 0 (zero) — Refers to the non-energy goods, except non-energy water. So,

> K_0 = capital for the production of non-energy goods, except the capital related to the non-energy water.
>
> L_0 = labor allocated to the production of non-energy goods except the workers related to the non-energy water.
>
> I_0 = investment for the accumulation and restoration of capital K_0.
>
> μ_0 =depreciation rate of capital K_0.

b) W_1 — Refers to the non-energy water. So,

> F_1 = production function for the extraction of resource W_1.
>
> K_1 = capital for the production of the extraction of resource W_1.

L_1 = labor for the production of the extraction of resource W_1.

I_1 = investment for the accumulation and restoration of capital K_1.

μ_1 = depreciation rate of capital K_1.

c) W_2 — Refers to the energy water, excluding water with a head. So,

W_2 = annual consumption or extraction rate of energy water (excluding hydroelectric water).

F_2 = production function for energy water.

K_2 = capital for the production of energy water.

L_2 = labor for the production of energy water.

I_2 = investment for the accumulation and restoration of capital K_2.

μ_2 = depreciation rate of capital K_2.

c) W_3 — Refers to the hydroelectric power energy water (water with a head). So,

W_3 = annual consumption or extraction rate of water with a head.

F_3 = production function for hydroelectric power.

K_3 = capital for the production of hydroelectric power.

L_3 = labor for the production of hydroelectric power.

I_3 = investment for the accumulation and restoration of capital K_3.

μ_3 = depreciation rate of capital K_2.

d) E — Refers to the energy resources (oil, coal, natural gas; typically fossil fuels). So,

E = annual consumption or extraction rate of energy resources (excluding hydroelectric).

F_4 = production function for the production of extraction of energy resources.

K_4 = capital for the production of extraction of energy resources.

L_4 = labor for the production of extraction of energy resources.

extraction of energy resource NR.

I_4 = investment for the accumulation and restoration of capital K_4.

μ_4 = depreciation rate of capital K_4.

D_E = deposits (natural reserves) of energy resource NR.

4.4.1 The Income Identity

$$F(K_0, L_0, E_N, W_1) = I_0 + I_1 + I_2 + I_3 + I_4 + Lc$$

It is implicit that the market is in equilibrium and that all the non-energy production is either consumed or invested.

4.4.2 The Investment Identity

The gross investment identity in the economy as a whole, except for the production of energy an water, is given by:

$$\frac{dK_0}{dt} = -\mu_0 K_0 + I_0$$

and represent the fact that investment is used to increase the capital stock K_0 and to restore the depreciated capital K_0. depreciation was neglected; one is thinking about a medium term period. Depreciation can be introduced easily in the model, but that is not the focus here. So, all capitals depreciation rates were neglected in the model.

For the production of non-energy water (W_1), one has:

$$\frac{dK_1}{dt} = -\mu_1 K_1 + \mu_{31} K_3 + \mu_{41} K_4 + I_1$$

where μ_{31} is the amount of hydroelectric energy capital used to increase the non-energy water capital K_1 and to restore the depreciated capital K_1, and μ_{41} is the amount of energy E (excluding hydroelectric) capital used to increase the non energy water W_1 capital K_1.

For the production of energy water (excluding hydroelectric) (W_2), one has:

$$\frac{dK_2}{dt} = -\mu_2 K_2 + \mu_{32} K_3 + \mu_{42} K_4 + I_2$$

where μ_{32} is the amount of hydroelectric energy W_3 capital used to increase the energy water W_2 (excluding hydroelectric) capital K_2, and μ_{42} is the amount of energy E (excluding hydroelectric) capital used to increase the energy water W_2 (excluding hydroelectric) capital K_2.

For the hydroelectric energy, the capital growth will depend only upon the respective investment:

$$\frac{dK_3}{dt} = -\mu_3 K_3 + I_3.$$

For the non-hydroelectric energy (E), the investment identity will be:

$$\frac{dK_4}{dt} = -\mu_4 K_4 + \mu_{24} K_2 + I_4,$$

where μ_{24} is the amount of energy water W_2 (excluding hydroelectric) capital K_2 used to increase non-hydroelectric energy E capital K_4.

4.4.3 Production Technologies

It is assumed that the markets for energy and the water are in equilibrium. Furthermore, all the energy and water extracted is supposedly consumed in the production of non-energy goods.

The production functions will be:

$$\begin{aligned} W_1 &= F_1(K_1, L_1)h_1(D) \\ W_2 &= F_2(K_2, L_2)h_2(D) \\ W_3 &= F_3(K_3, L_3)h_3(D) \\ E &= F_4(K_4, L_4)h_4(D_E) \end{aligned}$$

The water extraction production technology for the production of non-energy goods represents the means of extraction and making available water for this end. The irrigation process, the conveying of water for a industry that uses it heavily for cooling, water for fish or shrimp harbors, etc., are examples of this, as well as human and animal consumption. For such technology, as with all the others, one will also adopt the Hicks' neutrality assumption.

These production functions will, as usual, presumed to be continuous, concave, twice differentiable, monotonically nondecreasing, and, to some degree, homogeneous (depending on the technology).

The h functions generally decrease with D or D_E , this being valid for all restrained, limited, resource. They represent the exhaustion of the resources and the fact that when the resources diminish it is necessary to allocate more capital and more labor for the extraction.

So, the smaller the reserves, smaller h will be, and greater will be the effort for the extraction of the dwindling energy resource. These functions then represent the decreasing returns when used "mines" are explored, and an energy source is exhausting itself. It can be assumed in general that:

$$\lim_{D_i \to 0} h_i(D_i) = 0; \qquad \lim_{D_i \to \infty} h_i(D_i) = 1;$$

$$\lim_{D_i \to 0} \frac{dh_i(D_i)}{dD_i} = \infty; \qquad \lim_{D_i \to \infty} \frac{dh_i(D_i)}{dD_i} = 0;$$

Note that the resources only cease to exist in the limit or in the infinity. So, along the path, the resources can be treated as unlimited. The important thing here is to characterize the degree of exhaustion (or exploitation). For infinite reserve resources, like solar, aeolic, nuclear (fusion), etc., energy, $h = 1$, that is, all the effort is made by the technology used for the extraction. The hydroelectric case is similar to any exhaustible resource. This is the case because when there is a diminishing of the resources due to other uses, that is, when the availability of cubic meters per second of water for generation of energy is reduced, then, theoretically, according to the hypothesis, it is necessary to allocate more capital and more labor to generate more energy with less available water. In sum: when D decreases, the amount of resources used for the extraction should be greater in order to compensate this loss, and thus, h will decrease.

4.4.4 Reserve Consumption Dynamics

The consumption rate of the reserve D_E of energy resources will be given by:

$$\frac{dD_E}{dt} = -E.$$

It should be noticed that D_E includes all the energy resources, excluding hydroelectric power. Oil, natural gas, coal, charcoal, biomass (biofuels, like alcohol, biodiesel, and so on), firewood, wood fuel, uranium, etc., are all in this large aggregate.

The consumption rate of the reserve D of water resources will be given by:

$$\frac{dD}{dt} = -(W_1 + W_2 + W_3).$$

The total extraction rate of water resources reserve D is then the sum of three parcels: W_1, W_2, and W_3. Note that this equation also expresses the hypothesis that the system is in its tradeoff limit, that is, any amount of water taken away to the production of non-energy goods will affect the hydroelectric energy production and the energy water (excluding hydroelectric). And so on.

Note that when water is used to irrigate plantations for the production of crops that will be transformed into fuel, or firewood, or wood fuel, this water is of W_2 type.

4.4.5 Energy Balance

The net energy, E_N, to be used for the production of non energy goods (and non energy water) will be given by:

$$E_N = E + W_3 - W_2 - W_1.$$

4.4.6 The Labor Force Balance

Naturally, the following totalization relation should be imposed:

$$L = L_0 + L_1 + L_2 + L_3 + L_4.$$

4.4.7 Objective Functional

The assumed utilitarian structure will be represented here by an objective functional in the form of an intertemporal utility given by:

$$J = \int_0^\infty e^{-\delta t} L u(c) dt$$

As far as the function u is concerned, the common assumptions will be made, namely:

(a) It is continuous in $\mathbb{R}^+$.

(b) It is homogeneous of 1st degree ($u(\lambda c) = \lambda u(c)$).

(c) It is strictly concave in $\mathbb{R}^+$ ($u(\lambda c_1 + (1-\lambda)c_2) > \lambda u(c_1) + (1-\lambda)u(c_2)$, $\forall\ \lambda \in [0,1]$).

(d) It is monotonically increasing in $\mathbb{R}^+$ ($\frac{du}{dc} > 0$).

(e) It is of class C_2 in $\mathbb{R}^{++}$, that is, its derivative exists and is continuous in $\mathbb{R}^{++}$.

(f) The limits

$$\lim_{c \to 0} \frac{du}{dc} = +\infty, \qquad \lim_{c \to +\infty} \frac{du}{dc} = 0,$$

should be valid.

These hypothesis are basic. In the specialized literature [31–33, 35], authors make explicit use of them.

4.5 Assumptions of the Model

The basic approach adopted in this model is to consider the utilitarian structure, and exponential capital depreciation. The labor force growth dynamics, and the per-capita energy consumption are not considered here. See [24] for some results.

In this model, the workforce is the population, and this means that there is no unemployment and the Not Economically Active Population (NEAP) is not considered. Consumption distribution among the labor force is not modeled — an average is used.

It is assumed that people are indifferent between saving the money for the future generation and spend the money in consumption, as long as they have an interest rate of discount to compensate the action.

4.6 Problem Synthesis

The problem of the interaction between energy, water, and economy is, then, the optimal economic growth model formulated as:

$$\underset{I_0,I_1,I_2,I_3,I_4,E,W_3,W_1}{\text{Max}} J = \int_0^{\infty} e^{-\delta t} Lu(c)dt \tag{4.10}$$

subject to:

$$F(K_0, L_0, E_N, W_1) = I_0 + I_1 + I_2 + I_3 + I_4 + Lc \tag{4.11}$$

$$\frac{dK_0}{dt} = -\mu_0 K_0 + I_0 \tag{4.12}$$

$$\frac{dK_1}{dt} = -\mu_1 K_1 + \mu_{31} K_3 + \mu_{41} K_4 + I_1 \tag{4.13}$$

$$\frac{dK_2}{dt} = -\mu_2 K_2 + \mu_{32} K_3 + \mu_{42} K_4 + I_2 \tag{4.14}$$

$$\frac{dK_3}{dt} = -\mu_3 K_3 + I_3 \tag{4.15}$$

$$\frac{dK_4}{dt} = -\mu_4 K_4 + \mu_{24} K_2 + I_4 \tag{4.16}$$

$$W_1 = F_1(K_1, L_1)h_1(D) \tag{4.17}$$

$$W_2 = F_2(K_2, L_2)h_2(D) \tag{4.18}$$

$$W_3 = F_3(K_3, L_3)h_3(D) \tag{4.19}$$

$$E = F_4(K_4, L_4)h_4(D_E) \tag{4.20}$$

$$\frac{dD_E}{dt} = -E \tag{4.21}$$

$$\frac{dD}{dt} = -(W_1 + W_2 + W_3) \tag{4.22}$$

$$E_N = E + W_3 - W_2 - W_1 \tag{4.23}$$

$$L = L_0 + L_1 + L_2 + L_3 + L_4 \tag{4.24}$$

The model uses only one production function (F) for general non-energy goods, excluding non-energy water, a production function (F_1) for the production of extraction of non-energy water, a production function for the energy-water (excluding water for hydropower) (F_2), a production function (F_3) for hydropower water, and a production function for energy (excluding hydropower) (F_4), being the two sources of energy (E, W_3) interchangeable, in many cases, for the production of non-energy goods (and direct consumption, if the per-capita energy consumption were considered — typically as a control variable).

The state variables are: K_0, K_1, K_2, K_3, K_4, D_E and D.

The control forces are: I_0, I_1, I_2, I_3, I_4, E, W_3 and W_1.

The variables E_N, W_2, and c will depend upon the state and the control.

4.7 Results

The results of the model are collected here. The proofs are in the Appendix to this chapter. The necessary conditions expressed in these results should be the guide for policies concerning the energy-water nexus. In the optimum equilibrium one should have:

4.7.1 First Result: All capital shadow prices are equal

$$q_0 = q_1 = q_2 = q_3 = q_4 = q = \frac{du}{dc}, \tag{4.25}$$

that is, all capital shadow prices are equal, and have so a common value $q = \frac{du}{dc}$. The marginal utility of consuming non-energy goods should equal the capital accumulation rate, a classical result.

4.7.2 Second Result: The shadow prices of energy and water are equal

$$p_E = p_D = p. \tag{4.26}$$

that is, the shadow prices of energy (aggregate, excluding hydroelectric energy), E, and water $(W_1 + W_2 + W_3)$ are equal; they have a common value p.

4.7.3 Third Result: The energy internal price should be equal to the substitution rate between non-energy and energy goods

$$\frac{\partial F}{\partial E_N} = \frac{p}{q}. \tag{4.27}$$

4.7.4 Fourth Result: The contribution of W_1 to the total product is twice the contribution of E_N

$$\frac{\partial F}{\partial W_1} = 2 \cdot \frac{\partial F}{\partial E_N} = 2 \cdot \frac{p}{q}, \quad \text{and so:} \quad \frac{\frac{\partial F}{\partial W_1}}{\frac{\partial F}{\partial E_N}} = 2. \tag{4.28}$$

The contribution of W_1 to the total product (excluding energy and water) is twice the contribution of E_N. This result is already known [24]. It was extended for the case of using commodities as energy sources [25].

This result stresses the importance of non-energy water as an economic input for the economy as a whole.

4.7.5 Fifth Result: The contribution of W_1 to the total product is twice the contribution of E

$$\frac{\partial F}{\partial W_1} = 2\frac{\partial F}{\partial E} = 2\frac{p}{q}, \tag{4.29}$$

The contribution of W_1 to the total product (excluding energy and water) is twice the contribution of E. This result also highlights the importance of non-energy water.

4.7.6 Sixth Result: The contribution of W_1 to the total product is twice the contribution of W_3

$$\frac{\partial F}{\partial W_1} = 2\frac{\partial F}{\partial W_3} = 2\frac{p}{q}, \tag{4.30}$$

The contribution of W_1 to the total product (excluding energy and water) is twice the contribution of W_3. This result also highlights the importance of non-energy water, especially in the case of Brazil, which is very dependent of hydroelectric power.

4.7.7 Seventh Result: The in situ stock value should grow at the interest rate(The Hotelling's rule).

$$\frac{\dot{p}}{p} = \delta. \tag{4.31}$$

The in situ stock value should grow at the interest rate. This is precisely the Hotelling's rule [36]. Since water and energy (excluding solar, wind, sea wave, and tidal) are exhaustible, their common value (the shadow price p) will be given by:

$$p(t) = p(0)e^{\delta t}.$$

The shrinking of the reserves implies an exponential growth in their shadow price, at a rate equal to the interest rate. As the energy sources dwindle, it is imperative to start switching to solar, wind, sea wave, and tidal energy. Nuclear energy, although is in some sense non-exhaustible (especially if one dominates the nuclear fusion process), has the disadvantage of requiring lots of water for cooling, besides the water used in the turbine. And there are also the environmental (thermal pollution, etc.) and safety problems.

4.7.8 Eighth Result: Whatever the dynamics of D, of the water natural reserves, its impact on the growth of W_2 should be null

$$\frac{\partial W_2}{\partial D} = 0. \tag{4.32}$$

4.7.9 Ninth Result:The capital K_0 depreciation rate μ_0 of the economy as a whole should be larger than the non-energy water capital K_1 depreciation rate

$$\mu_0 > \mu_1. \tag{4.33}$$

The capital K_0 depreciation rate μ_0 of the economy as a whole (excluding energy goods and water) should be larger than the non-energy water capital K_1 depreciation rate. This means that the one should have a very robust (durable) technology for the production of non-energy water W_1.

4.7.10 Tenth Result: One limitation of the capital rent of K_2

$$\frac{\partial W_2}{\partial K_2} < \frac{q}{2p}(\mu_0 - \mu_2 + \mu_{24}). \tag{4.34}$$

The capital rent of K_2 is bounded above.

4.7.11 Eleventh Result: One limitation of the depreciation rate μ_0

$$\mu_0 > \max\{\mu_2 - \mu_{24}, \mu_3 - \mu_{31} - \mu_{32}, \mu_4 - \mu_{41} - \mu_{42}\}. \tag{4.35}$$

The hydroelectric power sector has an advanced, well established, and stabilized technology. The depreciation rate μ_3 is already very small.

4.7.12 Twelfth Result:Basis of equality for the capital rent of K_2

$$\frac{\partial W_2}{\partial K_2} = \frac{q}{2p}(\mu_1 - \mu_2 + \mu_{24}) = \frac{q}{2p}(\mu_3 - \mu_2 + \mu_{24} - \mu_{31} - \mu_{32}) = \frac{q}{2p}(\mu_4 - \mu_2 + \mu_{24} - \mu_{41} - \mu_{42}). \tag{4.36}$$

4.7.13 Thirteenth Result: Basis of equality for the depreciation rates

$$\mu_1 - \mu_2 = \mu_3 - \mu_2 - \mu_{31} - \mu_{32} = \mu_4 - \mu_2 - \mu_{41} - \mu_{42}. \tag{4.37}$$

4.7.14 Fourteenth Result:One limitation of the depreciation rate μ_1

$$\mu_1 > \mu_2 - \mu_{24} \tag{4.38}$$

4.7.15 Fifteenth Result:One limitation of the depreciation rate μ_3

$$\mu_3 > \mu_2 - \mu_{24} + \mu_{31} + \mu_{32} \tag{4.39}$$

4.7.16 Sixteenth Result:One limitation of the depreciation rate μ_4

$$\mu_4 > \mu_2 - \mu_{24} + \mu_{41} + \mu_{42} \tag{4.40}$$

4.7.17 Seventeenth Result: An increase in the K_2 capital rent will provoke a decrease in the K_0 capital rent

$$\frac{\partial W_2}{\partial K_2} \nearrow \Rightarrow \frac{\partial F}{\partial K_0} \searrow . \tag{4.41}$$

An increase in the K_2 capital rent will provoke a decrease in the K_0 capital rent; the marginal productivity of capital K_0 (with respect to F).

4.7.18 Eighteenth Result: In order to improve the K_0 capital rent, one could increase, for instance, the parameter μ_{24}

$$\mu_{24} \nearrow \Rightarrow \frac{\partial F}{\partial K_0} \nearrow . \tag{4.42}$$

In order to improve the K_0 capital rent, one could increase, for instance, the parameter μ_{24}, thus improving the contribution of a minimum capital K_2 to the energy capital K_4 growth ($\frac{dK_4}{dt}$). In other words, one should use less energy water (excluding hydroelectric), W_2, to produce the same amount of energy (E). This means a technological development, including better cycles, better cooling systems (perhaps hybrid water-air systems), and so on. And, of course, this means using less biomass for energy production (alcohol for fuel, for instance), since it requires lots of energy water (W_2).

4.7.19 Nineteenth Result: The capital K_2 depreciation rate μ_2 should be kept as small as possible

$$\mu_2 \nearrow \Rightarrow \frac{\partial F}{\partial K_0} \searrow . \tag{4.43}$$

The capital K_2 depreciation rate μ_2 should be kept as small as possible, in order not to negatively impact the K_0 capital rent. The technology for producing W_2 should be durable.

4.7.20 Twentieth Result: Investing in capital K_2 and allocating labor to produce energy-water W_2 has a negative effect in the economy as a whole

$$\frac{\partial F}{\partial K_2} < 0; \qquad \frac{\partial F}{\partial L_2} < 0. \tag{4.44}$$

Investing in capital K_2 and allocating labor to produce energy-water W_2 has a negative effect in the economy as a whole. This means that one should be switching as soon as possible to energy sources that do not require water for their operation, like solar, wind, sea wave, and tidal energy. Perhaps geisers fall in this category. One should strive to have $W_2 = 0$. In this case, this signifies abandoning all fossil fuels, coal, charcoal, biomass, and nuclear energy. In a short term it is hard to see a world like this, since one has to produce fuels to feed the cars, trucks, ships, trains, and airplanes. The alternative would be electric cars, buses, and trains. For water navigation one would have sailing boats or ships, aided by solar energy. For aircrafts the situation is even more acute: gliders, aided by solar energy. In a foreseeable time ahead, it is very unlike to get rid of fuels. They are siamese twins with transport. It is more realistic, so, to try to keep W_2 at a minimum. This means that one should promote technology development in the production function F_4.

4.7.21 Twenty First Result: The effort in producing non-energy water (W_1), hydroelectric energy (W_3), and energy (E), has a positive effect in the economy as a whole.

$$\frac{\partial F}{\partial K_1} > 0, \quad \frac{\partial F}{\partial L_1} > 0; \quad \frac{\partial F}{\partial K_3} > 0, \quad \frac{\partial F}{\partial L_3} > 0; \quad \frac{\partial F}{\partial K_4} > 0, \quad \frac{\partial F}{\partial L_4} > 0. \tag{4.45}$$

The effort in producing non-energy water (W_1), hydroelectric energy (W_3), and energy (excluding hydroelectric energy) (E), has a positive effect in the economy as a whole.

4.8 Conclusions

The results suggest that it is very important to have the best technology available for producing water not used for energy, and also to keep developing it. Water is by far the more important input to the economy, as compared to energy.

We should avoid as much as possible technologies for producing energy that use water. This means that we should be using solar energy, wind energy, wave or tidal power, and the like.

The economics rationale embedded in the model implies that we should be redirecting energy policies around the world. The classical ways of producing energy, including nuclear energy, imply the use of water (for cooling, etc.), which is a scarce resource.

As the population grows, water will be ever scarcer. And it cannot be substituted. It will be harder and harder to make water, a precious liquid, available to the population and the economy. More and more energy will be needed. The case of sea water desalination is illustrative; reverse osmosis requires lots of energy. Energy, which in its turn, will need more and more water for its production. The only way to escape from this trap is to improve the technology for producing energy (fuels, thermoelectric, for instance) using less water and use progressively more primary energy resources that do not need water in order to produce usable energy.

CHAPTER 5

Conclusion and Suggestions For Future Studies

This work presented two mathematical modeling to energy-water nexus analyses.

As the population grows, water will be more and more scarce. And it cannot be substituted. It will be harder and harder to make water, a precious liquid, available to the population and its economy. More and more energy will be needed.

The Two models proposed works well and can shortly represent energy-water nexus and their evolution, tendencies and scarcity prevent.

The only way to escape from this trap is to improve the technology for producing energy (fuels, thermoelectric, for instance) using less water, on one hand, and use progressively more energy primary sources that do not need water in order to produce usable energy, on the other hand.

5.0.1 The Energy-Water-Food-Transport Nexus

Energy, water, food and transport are inextricably linked, a fact that is increasingly recognized as one of the most important issues facing an economy.

Energy production needs water for the purposes of cooling and conversely, treating and transporting water takes energy. Additionally, conventional food production and distribution, which includes transportation — including the production and transportation of artificial fertilizers and pesticides — requires a tremendous amount of energy. So a shortage of water can severely inhibit both energy and food production, and may require additional energy to pump water over greater distances and from deeper wells for both drinking and irrigation. Labor force and people in general have to be transported, and this requires energy. Transportation systems (trains, trucks, buses, underground, tramways, street cars, ships, arplanes, cars, etc.) need energy. Plenty of it!

It is essential that a country that seeks to be healthy, secure and sustainable requires a holistic approach to the social, environmental and security challenges presented by the interdependencies between these four issues.

Food, transport, water and energy — they may not seem like they're connected but the systems that help produce and bring fresh food, transport and energy as well as clean, abundant water to all people, are intertwined. It takes water to create food and energy, it takes energy to move and treat water, to move people and goods, and to produce food, and sometimes food is used as a source of energy. Industrial agriculture, for example, is incredibly water intensive. These systems have become increasingly more complex and dependent upon one another. As a result, a disturbance in one system can wreak havoc in the others, so it's important to achieve a sustainable balance between the four.

Today, both business and government are constantly thinking about how to feed and transport more people, transport more goods, power more homes and cars, and provide clean drinking water. But it is increasingly apparent that the use of our precious resources to meet one need is inherently linked to the the others in the food, transport, water and energy "'nexus."

So how do we weigh the needs of agriculture, energy production and water use?

One has to make tough decisions about water, food and energy in the face of climate change, population growth, and economic pressures. The World Economic Forum (WEF) in Switzerland this year highlighted the need to shift current "silo thinking" about food, water or energy individually to a "nexus approach" which takes into account all three issues at once. Transportation is also part of the issue.

We need a systematic approach to solve the interconnected issues that link energy, water, food, and climate change. These sectors food and transport can be included in the general structure of the dynamic input-output Leontief model used to analyze the energy-water nexus.

5.0.2 The Future Energy-Water-Food-Transport Model

In the case of two or more products (energy, water, and food, for instance), whatever be the productive chain structure, the solution has the same "physiognomy":

$$\boldsymbol{x}(t) = e^{\mathcal{A}t}\boldsymbol{x}(0) + \int_0^t e^{\mathcal{A}(t-\tau)}\beta\boldsymbol{d}(\tau)d\tau,$$

now $e^{\mathcal{A}t}$ being a matrix. Note that the number of vector x coordinates is the same as in vectors d and s; this correspond to the n products. Thus $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{D}$ are all square $n \times n$ matrices.

In the "real world" the demand for the various products do not vary in a fixed proportion among them. So, for instance, d_1 could increase and d_2 decrease. This could happen depending on several factors. In the case of energy and water, for instance, one could have technological development, as well as production arrangements and public policies innovations, which in general will interfere (sometimes desirably) in this correlation. Thus, if the matrices K and B were chosen to be diagonal, one would be "tied" to the "physical" structure of the supply chain, with very low possibility of optimizing the chain as a whole. Both, K and B should be chosen full, that is, all their elements should be non null, so one could be able to accommodate all possible correlations between the demands for the various products, being these either positive or negative, whatever being theirs variances. This means that both, the production planning (policy) (K) and the stock control (B) should be established for the chain as a whole, as a function of the vectorial demand. In the case of the energy-water nexus this is crucial.

The numerical solution for the $e^{\mathcal{A}t}$ matrix is straight forward. One can use the Taylor series approximation for the simulations.

5.0.3 Integrated Policies for Energy, Water, Food, and Transport

The model structure allows the inclusion of any number of sectors of the economy, as well as the national income accounts, and a general equilibrium analysis. It is a natural setup for these studies, and thus a feasible tool for proposing national development policies.

Here one has four sectors that are more closely linked. One has then four state variables. This model was also implemented in a spreadsheet. The results are…

5.1 Suggestions For Future Studies

Earth and the communities that live upon it are part of a system. By approaching these massive problems from an integrated standpoint, one begins to solve problems in a more systematic way.

The energy-water nexus is gaining traction with diverse stakeholders around the world and it is becoming increasingly clear that one cannot plan for the planet's future if one does not consider energy and water together.

There is a close connection, or nexus, between energy and water:

It takes a significant amount of water to create energy. Water is used to cool steam electric power plants — fueled by coal, oil, natural gas and nuclear power — and is required to generate hydropower. Water is also used in great quantities during fuel extraction, refining and production.

It takes a significant amount of energy to extract, move and treat water for drinking and irrigation. It is used in the collection, treatment and disposal of waste water. Energy is also consumed when water is used by households and industry, especially through heating and cooling.

Water and energy policy, planning and management must be integrated to encourage conservation, motivate innovation and ensure sustainable use of water and energy.

It takes a significant amount of water to create energy, and a significant amount of energy to move and treat water.

You may not realize it, but when you use energy, you're also using water indirectly — lots of it!

Energy and water are fundamentally intertwined, but the linkages of these two vital resources also greatly impact the food and transportation sectors. More and more we hear about the energy-water-food nexus and how we are going to manage growing populations and the demands on all three resources. Plus, when we take into account the global impacts from climate change, we're looking at some big challenges — but also some incredible opportunities.

CHAPTER 6

Appendix A: An Input-Output Dynamical Model

Appendix

6.1 The Overall Dynamical System

Equations 3.14 will now be algebraically manipulated in order to obtain an equation for $\frac{dx}{dt}$ and s, as a function of x and d. One will have:

$$\begin{aligned} s &= (I-A)x - Bu \\ &= (I-A)x - BK(d-s) \\ (I-BK)s &= (I-A)x - BKd \\ \therefore s &= (I-BK)^{-1}(I-A)x - (I-BK)^{-1}BKd \end{aligned}$$

The production time rate will be:

$$\begin{aligned} \frac{dx}{dt} &= K(d-s) \\ &= K\left[d - (I-BK)^{-1}(I-A)x + (I-BK)^{-1}BKd\right] \\ &= -K(I-BK)^{-1}(I-A)x + K\left[I + (I-BK)^{-1}BK\right]d \\ &= -K(I-BK)^{-1}(I-A)x + K(I-BK)^{-1}d \end{aligned}$$

since

$$\left[I + (I - BK)^{-1}BK\right] = (I - BK)^{-1}.$$

The closed loop control system will then be described by:

$$\frac{dx}{dt} = -K(I - BK)^{-1}(I - A)x + K(I - BK)^{-1}d \tag{6.1}$$

$$s = (I - BK)^{-1}(I - A)x - (I - BK)^{-1}BKd \tag{6.2}$$

In the equilibrium point, that is, when $\frac{dx}{dt} = 0$, one will have:

$$u = K(d - s) = 0 \quad \therefore \quad d = s,$$

as it should be. In fact, one has

$$\frac{dx}{dt} = -K(I - BK)^{-1}(I - A)x + K(I - BK)^{-1}d = 0,$$

$$\therefore -(I - BK)^{-1}(I - A)x + (I - BK)^{-1}d = 0$$

$$\therefore (I - BK)^{-1}(I - A)x = (I - BK)^{-1}d.$$

But since

$$\left[I + (I - BK)^{-1}BK\right] = (I - BK)^{-1} \Rightarrow (I - BK)^{-1}BK = (I - BK)^{-1} - I,$$

it follows that, in the equilibrium point:

$$\begin{aligned} s &= (I - BK)^{-1}(I - A)x - (I - BK)^{-1}BKd \\ &= (I - BK)^{-1}d - (I - BK)^{-1}BKd \\ &= (I - BK)^{-1}d - \left[(I - BK)^{-1} - I\right]d \\ &= \left[(I - BK)^{-1} - (I - BK)^{-1} + I\right]d \\ &= Id \\ &= d. \end{aligned}$$

as it should be, with no restrictions whatsoever on the model matrices.

The restrictions on the model matrices should be though introduced as to guarantee stability. From the closed loop equation it can be observed that it is the demand that "controls" the system. To guarantee stability, it is necessary that the real part of the eigenvalues of the matrix

$$-K(I - BK)^{-1}(I - A)$$

be negative. In other words, the roots of the polynomial

$$\det\left[\lambda I + K(I - BK)^{-1}(I - A)\right] = 0, \tag{6.3}$$

where λ is the vector of eigenvalues, should have negative real parts. Complex roots should be avoided, since this would mean oscillations in the produced quantities.

The model allows so a great flexibility as far as the choices of the K and B matrices are concerned. Matrix A is typically given; it is, so to speak, "physical", "structural". By chosing K and B one could substantially alter a production chain dynamics, bringing it to a trajectory that optimizes a pre-established objective functional, defined by the chain managers.

Note that when the stock growth is negligible, one could set $B = 0$. In this case the model would be:

$$\frac{dx}{dt} = -K(I - A)x + Kd$$
$$s = (I - A)x$$

The matrix $-K(I - A)$ represents the proper dynamics of the system.

One should have in mind that the matrix $(I - A)$ must be always feasible, as mentioned before.

6.1.1 The Use of The Model

To run the model one has to have estimates of the elements of the A matrix (the existent "physical" structure), chose the K and B matrices, a prediction for the demand time series, and solve the linear differential equations system:

$$\frac{dx}{dt} = -K(I - BK)^{-1}(I - A)x + K(I - BK)^{-1}d \tag{6.4}$$

$$s = (I - BK)^{-1}(I - A)x - (I - BK)^{-1}BKd \tag{6.5}$$

Letting

$$\begin{aligned}
\mathcal{A} &= -K(I - BK)^{-1}(I - A) \\
\mathcal{B} &= K(I - BK)^{-1} \\
\mathcal{C} &= (I - BK)^{-1}(I - A) \\
\mathcal{D} &= -(I - BK)^{-1}BK
\end{aligned} \tag{6.6}$$

the linear dynamical system will then be described by:

$$\begin{aligned}
\frac{dx}{dt} &= \mathcal{A}x + \mathcal{B}d \\
s &= \mathcal{C}x + \mathcal{D}d
\end{aligned} \tag{6.7}$$

where

- x is the state variable;
- d is the control variable (the input);
- s is the system's output.

The simplest "production chain" (say, only energy, or only water) is the unitary chain. In this case one has only one product: x, which now is a scalar. The same is true for d and s. The parameters will also be scalars, and so the model of the system will be:

$$\frac{dx}{dt} = -\alpha x + \beta d \tag{6.8}$$

$$s = \gamma x + \delta d \tag{6.9}$$

If the fabrication of this unique product, x, at the initial time is $x(0)$, the production at any time t ahead will be

$$x(t) = e^{-\alpha t}x(0) + \int_0^t e^{-\alpha(t-\tau)}\beta d(\tau)d\tau \tag{6.10}$$

If the demand were constant, say,

$$d(t) = d,$$

the produced quantity would be:

$$x(t) = e^{-\alpha t}x(0) + \beta d\int_0^t e^{-\alpha(t-\tau)}d\tau = \frac{1-e^{-\alpha t}}{\alpha},$$

and thus, when quando $t \to \infty$, one would have:

$$\lim_{t\to\infty} x(t) = \frac{\beta d}{\alpha},$$

which, as a matter of fact, is the equilibrium point (when $\frac{dx}{dt} = 0$).

It happens in general that demand $d(t)$ is a stochastic process, and one has to consider numerical procedures to solve the equation, in this case. There will be no analytic expression for $d(t)$ which could be integrated. Once the series for $x(t)$ is obtained, the series for s is immediately calculated, by substituting x and d.

In the case of two or more products (energy, water, and food, for instance), whatever be the productive chain structure, the solution has the same "physiognomy":

$$\boldsymbol{x}(t) = e^{\mathcal{A}t}\boldsymbol{x}(0) + \int_0^t e^{\mathcal{A}(t-\tau)}\beta\boldsymbol{d}(\tau)d\tau,$$

now $e^{\mathcal{A}t}$ being a matrix. Note that the number of vector $\boldsymbol{x}$ coordinates is the same as in vectors $\boldsymbol{d}$ and $\boldsymbol{s}$; this correspond to the n products. Thus $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{D}$ are all square $n \times n$ matrices.

In the "real world" the demand for the various products do not vary in a fixed proportion among them. So, for instance, d_1 could increase and d_2 decrease. This could happen depending on several factors. In the case of energy and water, for instance, one could have technological development, as well as production arrangements and public policies innovations, which in general will interfere (sometimes desirably) in this correlation. Thus, if the matrices K and B were chosen to be diagonal, one would be "tied" to the "physical" structure of the supply chain (the energy-water nexus), with very low possibility of optimizing the chain as a whole. Both, K and B should be chosen full, that is, all their elements should be non null, so one could be able to accommodate all possible correlations between the demands for the various products, being these either positive or negative, whatever being theirs variances. This means that both, the production planning (policy) (K) and the stock control (B) should be established for the chain as a whole, as a function of the vectorial demand. In the case of the energy-water nexus this is crucial.

The numerical solution for the $e^{\mathcal{A}t}$ matrix is straightforward. One can use the Taylor series approximation for the simulations.

6.2 A Spreadsheet Implementation of The Model

A numerical implementation of the model was done using a spreadsheet, as was done in [9].

6.2.1 The Numerical Evaluation of a Dynamical Linear System State Transition Matrix

A Taylor series approximation was used to calculate the numerical solutions of the differential equations.

$$\phi(t) = e^{Ft} \approx \sum_{k=0}^{N} \frac{(Ft)^k}{k!}.$$

where F is a $n \times n$ square matrix.

$$e^{FT} \approx I + FT + \frac{(FT)^2}{2!} + \frac{(FT)^3}{3!} + \cdots + \frac{(FT)^{IPROX}}{IPROX!}.$$

In most applications one uses between 10 and 20 terms in the series.

$$\exp(F) = e^F = I + \frac{F}{1!} + \frac{F^2}{2!} + \frac{F^3}{3!} + \cdots + \frac{F^k}{k!} + \cdots = \sum_{k=0}^{\infty} \frac{F^k}{k!}.$$

6.3 Discretization of the Model

Consider the multidimensional linear dynamical system:

$$\dot{x} = \mathcal{A}x + \mathcal{B}u$$

One wants to discretize it, in order to make the calculations and simulations, and represent it by the equation:

$$x((k+1)T) = G(T)x(kT) + H(T)u(kT)$$

The matrices G and H will depend upon the sampling period T. Once this period is fixed, G and H will be constant matrices.

In the continuous case the solution is given by:

$$x(t) = e^{\mathcal{A}t}x(0) + e^{\mathcal{A}t}\int_0^t e^{-\mathcal{A}\tau}\mathcal{B}u(\tau)d\tau$$

One assumes that between any two consecutive sampling instants, all u components are constant, that is,

$$u(t) = u(kT)$$

for the k-th sampling period. Since

$$x((k+1)T) = e^{\mathcal{A}(k+1)T}x(0) + e^{\mathcal{A}(k+1)T}\int_0^{(k+1)T} e^{-\mathcal{A}\tau}\mathcal{B}u(\tau)d\tau$$

and

$$x(kT) = e^{\mathcal{A}kT}x(0) + e^{\mathcal{A}kT}\int_0^{kT} e^{\mathcal{A}\tau}\mathcal{B}u(\tau)d\tau$$

multiplying this last expression by $e^{\mathcal{A}T}$ and subtracting from the previous one, one obtains:

$$\begin{aligned} x((k+1)T) &= e^{\mathcal{A}T}x(kT) + e^{\mathcal{A}(k+1)T}\int_{kT}^{(k+1)T} e^{-\mathcal{A}\tau}\mathcal{B}u(\tau)d\tau \\ &= e^{\mathcal{A}T}x(kT) + e^{\mathcal{A}T}\int_0^T e^{-\mathcal{A}t}\mathcal{B}u(kT)dt = \\ &= e^{\mathcal{A}T}x(kT) + \int_0^T e^{-\mathcal{A}\lambda}\mathcal{B}u(kT)d\lambda \qquad (*) \end{aligned}$$

where $\lambda = T - t$. If one defines

$$\begin{cases} G(T) = e^{\mathcal{A}T} \\ H(T) = \left(\int_0^T e^{\mathcal{A}t} dt\right) \mathcal{B} \end{cases}$$

then equation $(*)$ becomes

$$x((k+1)T) = G(T)x(kT) + H(T)u(kT)$$

Simulations of the model for the case of the energy-water nexus were done using a spreadsheet. Two main cases were studied: separated (independent) production policies for energy and water, and integrated policies. The results are presented in the sequel.

6.4 The System Discretization Computations

$$\frac{d\vec{x}}{dt} = -K(I - BK)^{-1}(I - A)\vec{x} + K(I - BK)^{-1}\vec{d} \tag{6.11}$$

$$\vec{s} = (I - BK)^{-1}(I - A)\vec{x} - (I - BK)^{-1}BK\vec{d} \tag{6.12}$$

6.5 The Numerical Computations for the State Transition Matrix

$$e^{[-K(I-BK)^{-1}(I-A)]t} \approx \sum_{k=0}^{20} \frac{\{[-K(I - BK)^{-1}(I - A)]t\}^k}{k!}.$$

$$\int e^{[-K(I-BK)^{-1}(I-A)]t} dt \approx \sum_{k=0}^{20} \frac{[-K(I - BK)^{-1}(I - A)]^k t^{k+1}}{(k+1)!}.$$

For the discretization:

$$G(T) = e^{[-K(I-BK)^{-1}(I-A)]T} \approx \sum_{k=0}^{20} \frac{\{[-K(I - BK)^{-1}(I - A)]T\}^k}{k!} =$$
$$= I + [-K(I - BK)^{-1}(I - A)]T + \frac{\{[-K(I - BK)^{-1}(I - A)]T\}^2}{2!} +$$
$$+ \frac{\{[-K(I - BK)^{-1}(I - A)]T\}^3}{3!} + \cdots + \frac{\{[-K(I - BK)^{-1}(I - A)]T\}^{20}}{20!}.$$

$$H(T) = \left\{ \int_0^T e^{[-K(I-BK)^{-1}(I-A)]t} dt \right\} [K(I-BK)^{-1}] \approx$$

$$\approx \left\{ \sum_{k=0}^{20} \frac{\{[-K(I-BK)^{-1}(I-A)]T\}^{k+1}}{(k+1)!} \right\} \times [K(I-BK)^{-1}] =$$

$$= \left\{ [-K(I-BK)^{-1}(I-A)]T + \frac{\{[-K(I-BK)^{-1}(I-A)]T\}^2}{2!} + \right.$$

$$\left. + \frac{\{[-K(I-BK)^{-1}(I-A)]T\}^3}{3!} + \cdots + \frac{\{[-K(I-BK)^{-1}(I-A)]T\}^{21}}{21!} \right\} [K(I-BK)^{-1}].$$

The discrete simulation equation is then:

$$\vec{x}((k+1)T) = G(T)\vec{x}(kT) + H(T)\vec{d}(kT),\ k = 0, 1, 2, \cdots, 30.$$

In the simulation, the equation which computes the supply continues to be:

$$\vec{s}(kT) = (I-BK)^{-1}(I-A)\vec{x}(kT) - (I-BK)^{-1}BK\vec{d}(kT), \quad k = 0, 1, 2, \cdots, 30.$$

The value of T should be in a spreadsheet fixed cell. One can begin with $T = 1$.

CHAPTER 7

Appendix B: An Optimal Control Model

Appendix

The Necessary Optimality Conditions.

7.1 The Hamiltonian

The Hamiltonian will be given by:

$$H = e^{-\delta t}\{Lu(c) + q_0(-\mu_0 K_0 + I_0) + q_1(-\mu_1 K_1 + \mu_{31} K_3 + \mu_{41} K_4 + I_1) + \\ + q_2(-\mu_2 K_2 + \mu_{32} K_3 + \mu_{42} K_4 + I_2) + q_3(-\mu_3 K_3 + I_3) + \\ + q_4(-\mu_4 K_4 + \mu_{24} K_2 + I_4) + p_E(-E) + p_D[-(W_1 + W_2 + W_3)]\} \quad (7.1)$$

where $e^{-\delta t}q_0$, $e^{-\delta t}q_1$, $e^{-\delta t}q_2$, $e^{-\delta t}q_3$, $e^{-\delta t}q_4$, $e^{-\delta t}p_E$, and $e^{-\delta t}p_D$ are the costate variables.

7.2 Basic Computations

From (4.11) one obtains:

$$c = \frac{1}{L}[F(K_0, L_0, E_N, W_1) - I_0 - I_1 - I_2 - I_3 - I_4],$$

from which it follows that:

$$\frac{\partial c}{\partial F} = \frac{1}{L}; \quad \frac{\partial c}{\partial I_0} = \frac{\partial c}{\partial I_1} = \frac{\partial c}{\partial I_2} = \frac{\partial c}{\partial I_3} = \frac{\partial c}{\partial I_4} = -\frac{1}{L}.$$

From (4.23) one will have:

$$\frac{\partial E_N}{\partial E} = 1; \quad \frac{\partial E_N}{\partial W_3} = 1; \quad \frac{\partial E_N}{\partial W_2} = -1; \quad \frac{\partial E_N}{\partial W_1} = -1.$$

7.3 Necessary Conditions for the Maximization of the Hamiltonian

1. $\frac{\partial H}{\partial I_i} = 0 \Rightarrow \frac{\partial H}{\partial I_i} = e^{-\delta t}\left(L\frac{du}{dc}\frac{\partial c}{\partial I_i} + q_i\right) = 0$ and therefore

$$q_i = \frac{du}{dc}, \quad i = 0, 1, 2, 3, 4. \tag{7.2}$$

Thus, $q_0 = q_1 = q_2 = q_3 = q_4 = q$, where q is the common value of all capital shadow prices.

2. $\frac{\partial H}{\partial E} = 0 \Rightarrow \frac{\partial H}{\partial E} = e^{-\delta t}\left[L\frac{du}{dc}\frac{\partial c}{\partial F}\left(\frac{\partial F}{\partial E_N}\cdot\frac{\partial E_N}{\partial E}\right) - p_E\right] = e^{-\delta t}\left[\frac{du}{dc}\frac{\partial F}{\partial E_N} - p_E\right] = 0$
   $\therefore p_E = \frac{du}{dc}\cdot\frac{\partial F}{\partial E_N}, \quad \frac{\partial F}{\partial E_N} = \frac{p_E}{q}$, since $\frac{du}{dc} = q$.

3. $\frac{\partial H}{\partial W_3} = 0 \Rightarrow \frac{\partial H}{\partial W_3} = e^{-\delta t}\left[L\frac{du}{dc}\frac{\partial c}{\partial F}\left(\frac{\partial F}{\partial E_N}\cdot\frac{\partial E_N}{\partial W_3}\right) - p_D\right] = e^{-\delta t}\left[\frac{du}{dc}\frac{\partial F}{\partial E_N} - p_D\right] = 0$
   $\therefore p_D = \frac{du}{dc}\cdot\frac{\partial F}{\partial E_N}, \quad \frac{\partial F}{\partial E_N} = \frac{p_D}{q}$, since $\frac{du}{dc} = q$.

From this item and the immediately previous item, one concludes that $p_E = p_D$, and the common value will be called p. So:

$$\frac{\partial F}{E_N} = \frac{p}{q}. \tag{7.3}$$

4. $\frac{\partial H}{\partial W_1} = 0 \Rightarrow \frac{\partial H}{\partial W_1} = e^{-\delta t}\left[L\frac{du}{dc}\frac{\partial c}{\partial F}\left(\frac{\partial F}{\partial E_N}\cdot\frac{\partial E_N}{\partial W_1} + \frac{\partial F}{\partial W_1}\right) - p_D\right] =$
   $= e^{-\delta t}\left[\frac{du}{dc}\left(-\frac{\partial F}{\partial E_N} + \frac{\partial F}{\partial W_1}\right) - p_D\right] = 0$
   $\therefore p_D = \frac{du}{dc}\cdot\left(\frac{\partial F}{\partial W_1} - \frac{\partial F}{E_N}\right) = q\left(\frac{\partial F}{\partial W_1} - \frac{\partial F}{E_N}\right)$, , since $\frac{du}{dc} = q$, and so $\frac{\partial F}{\partial W_1} = \frac{p_D}{q} + \frac{\partial F}{E_N}$.

But, from the previous item, $\frac{\partial F}{E_N} = \frac{p_D}{q}$, and so one concludes that $\frac{\partial F}{\partial W_1} = 2\frac{\partial F}{E_N} = 2\frac{p_D}{q} = 2\frac{p}{q}$.

One has then:

$$\frac{\partial F}{\partial W_1} = 2\frac{\partial F}{\partial E_N} = 2\frac{p}{q}. \tag{7.4}$$

Since

$$\frac{\partial F}{\partial E} = \frac{\partial F}{\partial E_N}\frac{\partial E_N}{\partial E} = \frac{\partial F}{\partial E_N} \quad \text{and} \quad \frac{\partial F}{\partial W_3} = \frac{\partial F}{\partial E_N}\frac{\partial E_N}{\partial W_3} = \frac{\partial F}{\partial E_N}$$

one concludes that:

$$\frac{\partial F}{\partial W_1} = 2\frac{\partial F}{\partial E} = 2\frac{p}{q}, \tag{7.5}$$

and

$$\frac{\partial F}{\partial W_1} = 2\frac{\partial F}{\partial W_3} = 2\frac{p}{q}, \tag{7.6}$$

7.4 The Costate Variables Dynamics

1. $\frac{d\left(e^{-\delta t} q_0\right)}{dt} = -\frac{\partial H}{\partial K_0} \quad \therefore e^{-\delta t}\left(\frac{dq_0}{dt} - \delta q_0\right) = -e^{-\delta t}\left(-q_0\mu_0 + L\frac{du}{dc}\frac{\partial c}{\partial F}\frac{\partial F}{\partial K_0}\right)$
   $\therefore \frac{dq_0}{dt} = \delta q_0 + q_0\mu_0 - \frac{du}{dc}\cdot\frac{\partial F}{\partial K_0}$

2. $\frac{d\left(e^{-\delta t} q_1\right)}{dt} = -\frac{\partial H}{\partial K_1} \quad \therefore e^{-\delta t}\left(\frac{dq_1}{dt} - \delta q_1\right) =$
   $= -e^{-\delta t}\left[-q_1\mu_1 + L\frac{du}{dc}\frac{\partial c}{\partial F}\left(\frac{\partial F}{\partial E_N}\frac{\partial E_N}{\partial W_1}\frac{\partial W_1}{\partial K_1} + \frac{\partial F}{\partial W_1}\frac{\partial W_1}{\partial K_1}\right) - p_D\frac{\partial W_1}{\partial K_1}\right]$

$$\begin{aligned}
\therefore \frac{dq_1}{dt} &= \delta q_1 + q_1\mu_1 - \frac{du}{dc}\left(-\frac{\partial F}{\partial E_N}\frac{\partial W_1}{\partial K_1} + \frac{\partial F}{\partial W_1}\frac{\partial W_1}{\partial K_1}\right) + p_D\frac{\partial W_1}{\partial K_1}\\
&= \delta q_1 + q_1\mu_1 - \frac{du}{dc}\left(\frac{\partial F}{\partial W_1} - \frac{\partial F}{\partial E_N}\right)\frac{\partial W_1}{\partial K_1} + p_D\frac{\partial W_1}{\partial K_1}\\
&= \delta q_1 + q_1\mu_1 + \left[p_D - \frac{du}{dc}\left(\frac{\partial F}{\partial W_1} - \frac{\partial F}{\partial E_N}\right)\right]\frac{\partial W_1}{\partial K_1}\\
&= \delta q_1 + q_1\mu_1 + \left[p_D - q_1\left(\frac{\partial F}{\partial W_1} - \frac{\partial F}{\partial E_N}\right)\right]\frac{\partial W_1}{\partial K_1}
\end{aligned}$$

3. $\frac{d\left(e^{-\delta t} q_2\right)}{dt} = -\frac{\partial H}{\partial K_2} \quad \therefore e^{-\delta t}\left(\frac{dq_2}{dt} - \delta q_2\right) =$
   $-e^{-\delta t}\left(-q_2\mu_2 + L\frac{du}{dc}\frac{\partial c}{\partial F}\frac{\partial F}{\partial E_N}\frac{\partial E_N}{\partial W_2}\frac{\partial W_2}{\partial K_2} + \mu_{24}q_4 - p_D\frac{\partial W_2}{\partial K_2}\right)$

$$\begin{aligned}
\therefore \frac{dq_2}{dt} &= \delta q_2 + q_2\mu_2 - \mu_{24}q_4 + \frac{du}{dc}\cdot\frac{\partial F}{\partial E_N}\frac{\partial W_2}{\partial K_2} + p_D\frac{\partial W_2}{\partial K_2}\\
&= \delta q_2 + q_2\mu_2 - \mu_{24}q_4 + \left(p_D + \frac{du}{dc}\cdot\frac{\partial F}{\partial E_N}\right)\frac{\partial W_2}{\partial K_2}
\end{aligned}$$

4. $\frac{d\left(e^{-\delta t} q_3\right)}{dt} = -\frac{\partial H}{\partial K_3} \quad \therefore e^{-\delta t}\left(\frac{dq_3}{dt} - \delta q_3\right) =$
   $= -e^{-\delta t}\left(-q_3\mu_3 + L\frac{du}{dc}\frac{\partial c}{\partial F}\frac{\partial F}{\partial E_N}\frac{\partial E_N}{\partial W_3}\frac{\partial W_3}{\partial K_3} + \mu_{31}q_1 + \mu_{32}q_2 - p_D\frac{\partial W_3}{\partial K_3}\right)$

$$\begin{aligned}
\therefore \frac{dq_3}{dt} &= \delta q_3 + q_3\mu_3 - \mu_{31}q_1 - \mu_{32}q_2 - \frac{du}{dc}\frac{\partial F}{\partial E_N}\frac{\partial W_3}{\partial K_3} + p_D\frac{\partial W_3}{\partial K_3}\\
&= \delta q_3 + q_3\mu_3 - \mu_{31}q_1 - \mu_{32}q_2 + \left(p_D - \frac{du}{dc}\frac{\partial F}{\partial E_N}\right)\frac{\partial W_3}{\partial K_3}
\end{aligned}$$

5. $\frac{d\left(e^{-\delta t}q_4\right)}{dt} = -\frac{\partial H}{\partial K_4} \quad \therefore e^{-\delta t}\left(\frac{dq_4}{dt} - \delta q_4\right) =$
$= -e^{-\delta t}\left(-q_4\mu_4 + L\frac{du}{dc}\frac{\partial c}{\partial F}\frac{\partial F}{\partial E_N}\frac{\partial E_N}{\partial E}\frac{\partial E}{\partial K_4} + \mu_{41}q_1 + \mu_{42}q_2 - p_E\frac{\partial E}{\partial K_4}\right)$

$$\begin{aligned}\therefore \frac{dq_4}{dt} &= \delta q_4 + q_4\mu_4 - \frac{du}{dc}\cdot\frac{\partial F}{\partial E_N}\frac{\partial E}{\partial K_4} - \mu_{41}q_1 - \mu_{42}q_2 + p_E\frac{\partial E}{\partial K_4}\\ &= \delta q_4 + q_4\mu_4 - \mu_{41}q_1 - \mu_{42}q_2 - \frac{du}{dc}\cdot\frac{\partial F}{\partial E_N}\frac{\partial E}{\partial K_4} + p_E\frac{\partial E}{\partial K_4}\\ &= \delta q_4 + q_4\mu_4 - \mu_{41}q_1 - \mu_{42}q_2 + \left(p_E - \frac{du}{dc}\cdot\frac{\partial F}{\partial E_N}\right)\frac{\partial E}{\partial K_4}\end{aligned}$$

6. $\frac{d\left(e^{-\delta t}p_E\right)}{dt} = -\frac{\partial H}{\partial D_E} \quad \therefore e^{-\delta t}\left(\frac{dp_E}{dt} - \delta p_E\right) = -e^{-\delta t}\left(L\frac{du}{dc}\frac{\partial c}{\partial F}\frac{\partial F}{\partial E_N}\frac{\partial E_N}{\partial E}\frac{\partial E}{\partial D_E} - p_E\frac{\partial E}{\partial D_E}\right) =$
$= -e^{-\delta t}\left(\frac{du}{dc}\frac{\partial F}{\partial E_N}\frac{\partial E}{\partial D_E} - p_E\frac{\partial E}{\partial D_E}\right)$

$$\frac{dp_E}{dt} = \delta p_E + \left(p_E - \frac{du}{dc}\frac{\partial F}{\partial E_N}\right)\frac{\partial E}{\partial D_E}$$

7. $\frac{d\left(e^{-\delta t}p_D\right)}{dt} = -\frac{\partial H}{\partial D} \quad \therefore e^{-\delta t}\left(\frac{dp_D}{dt} - \delta p_D\right) =$
$= e^{-\delta t}\left\{L\frac{du}{dc}\frac{\partial c}{\partial F}\left[\frac{\partial F}{\partial E_N}\left(\frac{\partial E_N}{\partial W_1}\frac{\partial W_1}{\partial D} + \frac{\partial E_N}{\partial W_2}\frac{\partial W_2}{\partial D} + \frac{\partial E_N}{\partial W_3}\frac{\partial W_3}{\partial D}\right) + \frac{\partial F}{\partial W_1}\frac{\partial W_1}{\partial D}\right]\right\} -$
$-e^{-\delta t}\left\{-p_D\left(\frac{\partial W_1}{\partial D} + \frac{\partial W_2}{\partial D} + \frac{\partial W_3}{\partial D}\right)\right\} =$
$= -e^{-\delta t}\left\{\frac{du}{dc}\left[\frac{\partial F}{\partial E_N}\left(-\frac{\partial W_1}{\partial D} - \frac{\partial W_2}{\partial D} + \frac{\partial W_3}{\partial D}\right) + \frac{\partial F}{\partial W_1}\frac{\partial W_1}{\partial D}\right] - p_D\left(\frac{\partial W_1}{\partial D} + \frac{\partial W_2}{\partial D} + \frac{\partial W_3}{\partial D}\right)\right\} =$
$= -e^{-\delta t}\left\{-\frac{du}{dc}\frac{\partial F}{\partial E_N}\left(\frac{\partial W_1}{\partial D} + \frac{\partial W_2}{\partial D}\right) + \frac{du}{dc}\frac{\partial F}{\partial E_N}\frac{\partial W_3}{\partial D} + \frac{du}{dc}\frac{\partial F}{\partial W_1}\frac{\partial W_1}{\partial D} - p_D\left(\frac{\partial W_1}{\partial D} + \frac{\partial W_2}{\partial D}\right) - p_D\frac{\partial W_3}{\partial D}\right\}$

$$\therefore \frac{dp_D}{dt} = \delta p_D + \left(p_D + \frac{du}{dc}\frac{\partial F}{\partial E_N}\right)\left(\frac{\partial W_1}{\partial D} + \frac{\partial W_2}{\partial D}\right) + \left(p_D - \frac{du}{dc}\frac{\partial F}{\partial E_N}\right)\frac{\partial W_3}{\partial D} - \frac{du}{dc}\frac{\partial F}{\partial W_1}\frac{\partial W_1}{\partial D}$$

Since all p_i's are equal to the common value p and all q_j's are equal to the common value q, and $\frac{\partial F}{\partial E_N} = \frac{p}{q}$, and $\frac{\partial F}{\partial W_1} = 2\frac{p}{q}$ (section 7.3), one obtains from items 1, 2, 3, 4, 5, 6, and 7 of this section 7.4:

1′. $\frac{dq}{dt} = \left(\delta + \mu_0 - \frac{\partial F}{\partial K_0}\right)q \quad \therefore \frac{\dot{q}}{q} = \delta + \mu_0 - \frac{\partial F}{\partial K_0}$

2′. $\frac{dq}{dt} = \delta q + q\mu_1 + \left[p - q\left(2\frac{p}{q} - \frac{p}{q}\right)\right]\frac{\partial W_1}{\partial K_1} = (\delta + \mu_1)q \quad \therefore \frac{\dot{q}}{q} = \delta + \mu_1$

3′. $\frac{dq}{dt} = \delta q + q\mu_2 - \mu_{24}q + \left(p + q\cdot\frac{p}{q}\right)\frac{\partial W_2}{\partial K_2} = \delta q + q\mu_2 - \mu_{24}q + 2p\frac{\partial W_2}{\partial K_2} = \delta q + q\mu_2 - \mu_{24}q +$
$+q\frac{\partial F}{\partial W_1}\frac{\partial W_2}{\partial K_2} = \left(\delta + \mu_2 - \mu_{24} + \frac{\partial F}{\partial W_1}\frac{\partial W_2}{\partial K_2}\right)q \quad \therefore \frac{\dot{q}}{q} = \delta + \mu_2 - \mu_{24} + \frac{\partial F}{\partial W_1}\frac{\partial W_2}{\partial K_2}$

4′. $\frac{dq}{dt} = \delta q + q\mu_3 - \mu_{31}q - \mu_{32}q + \left(p - q\frac{p}{q}\right)\frac{\partial W_3}{\partial K_3} = (\delta + \mu_3 - \mu_{31} - \mu_{32})q \quad \therefore \frac{\dot{q}}{q} =$
$\delta + \mu_3 - \mu_{31} - \mu_{32}$

5′. $\frac{dq}{dt} = \delta q + q\mu_4 - \mu_{41}q - \mu_{42}q + \left(p - q\frac{p}{q}\right)\frac{\partial E}{\partial K_4} = (\delta + \mu_4 - \mu_{41} - \mu_{42})q \quad \therefore \frac{\dot{q}}{q} = \delta + \mu_4 - \mu_{41} - \mu_{42}$

6′. $\frac{dp}{dt} = \delta p + \left(p - q\frac{p}{q}\right)\frac{\partial E}{\partial D_E} = \delta p$ and therefore

$$\frac{\dot{p}}{p} = \delta. \tag{7.7}$$

7′. $\frac{dp}{dt} = \delta p + \left(p + q\frac{p}{q}\right)\left(\frac{\partial W_1}{\partial D} + \frac{\partial W_2}{\partial D}\right) + \left(p - q\frac{p}{q}\right)\frac{\partial W_3}{\partial D} - q2\frac{p}{q}\frac{\partial W_1}{\partial D} =$
$= \delta p + 2p\left(\frac{\partial W_1}{\partial D} + \frac{\partial W_2}{\partial D}\right) - 2p\frac{\partial W_1}{\partial D} = \delta p + 2p\frac{\partial W_2}{\partial D}$ and therefore

$$\frac{\dot{p}}{p} = \delta + 2p\frac{\partial W_2}{\partial D} \tag{7.8}$$

From 7.7 and 7.8 one obtains:

$$\frac{\partial W_2}{\partial D} = 0. \tag{7.9}$$

One concludes then that:

$$\frac{\dot{q}}{q} - \delta = \mu_0 - \frac{\partial F}{\partial K_0} = \mu_1 = \mu_2 - \mu_{24} + \frac{\partial F}{\partial W_1}\frac{\partial W_2}{\partial K_2} = \mu_3 - \mu_{31} - \mu_{32} = \mu_3 - \mu_{41} - \mu_{42}$$

$$\therefore \frac{\partial F}{\partial K_0} = \mu_0 - \mu_1 = \mu_0 - \mu_2 + \mu_{24} - \frac{\partial F}{\partial W_1}\frac{\partial W_2}{\partial K_2} = \mu_0 - \mu_3 + \mu_{31} + \mu_{32} = \mu_0 - \mu_4 + \mu_{41} + \mu_{42}$$

Since, by hypothesis, $\frac{\partial F}{\partial K_0} > 0$, one has to have:

$$\mu_0 > \mu_1 \tag{7.10}$$

By the same token

$$\mu_0 - \mu_2 + \mu_{24} - \frac{\partial F}{\partial W_1}\frac{\partial W_2}{\partial K_2} > 0,$$

and so

$$\frac{\partial W_2}{\partial K_2} < \frac{q}{2p}(\mu_0 - \mu_2 + \mu_{24}). \tag{7.11}$$

Since $\frac{\partial W_2}{\partial K_2} > 0$, one must have

$$\mu_0 > \mu_2 - \mu_{24}. \tag{7.12}$$

Also

$$\mu_0 > \mu_3 - \mu_{31} - \mu_{32}, \tag{7.13}$$

and

$$\mu_0 > \mu_4 - \mu_{41} - \mu_{42}, \tag{7.14}$$

and thus:

$$\mu_0 > \max\{\mu_2 - \mu_{24}, \mu_3 - \mu_{31} - \mu_{32}, \mu_4 - \mu_{41} - \mu_{42}\}. \tag{7.15}$$

From the equalities in the expression of $\frac{\partial F}{\partial K_0}$, one obtains

$$-\mu_1 = -\mu_2 + \mu_{24} - \frac{\partial F}{\partial W_1}\frac{\partial W_2}{\partial K_2}$$

and so

$$\frac{\partial W_2}{\partial K_2} = \frac{q}{2p}(\mu_1 - \mu_2 + \mu_{24}). \tag{7.16}$$

It is also true that:

$$\frac{\partial W_2}{\partial K_2} = \frac{q}{2p}(\mu_3 - \mu_2 + \mu_{24} - \mu_{31} - \mu_{32}), \tag{7.17}$$

and

$$\frac{\partial W_2}{\partial K_2} = \frac{q}{2p}(\mu_4 - \mu_2 + \mu_{24} - \mu_{41} - \mu_{42}). \tag{7.18}$$

Hence, since $\frac{\partial W_2}{\partial K_2} > 0$, one obtains:

$$\mu_1 > \mu_2 - \mu_{24} \tag{7.19}$$

$$\mu_3 > \mu_2 - \mu_{24} + \mu_{31} + \mu_{32} \tag{7.20}$$

$$\mu_4 > \mu_2 - \mu_{24} + \mu_{41} + \mu_{42} \tag{7.21}$$

The fact that

$$\frac{\partial F}{\partial K_0} = \mu_0 - \mu_2 + \mu_{24} - \frac{\partial F}{\partial W_1}\frac{\partial W_2}{\partial K_2} = \mu_0 - \mu_2 + \mu_{24} - \frac{2p}{q}\frac{\partial W_2}{\partial K_2}$$

implies that:

$$\frac{\partial W_2}{\partial K_2} \nearrow \Rightarrow \frac{\partial F}{\partial K_0} \searrow, \tag{7.22}$$

and that:

$$\mu_{24} \nearrow \Rightarrow \frac{\partial F}{\partial K_0} \nearrow, \tag{7.23}$$

and also that

$$\mu_2 \nearrow \Rightarrow \frac{\partial F}{\partial K_0} \searrow . \tag{7.24}$$

Note that

$$\begin{aligned}
\frac{\partial F}{\partial K_1} &= \frac{\partial F}{\partial E_N}\cdot\frac{\partial E_N}{\partial K_1}+\frac{\partial F}{\partial W_1}\cdot\frac{\partial W_1}{\partial K_1}=\frac{\partial F}{\partial E_N}\cdot\frac{\partial E_N}{\partial W_1}\cdot\frac{\partial W_1}{\partial K_1}\\
&=\frac{\partial F}{\partial E_N}\cdot(-1)\cdot\frac{\partial W_1}{\partial K_1}+\frac{\partial F}{\partial W_1}\cdot\frac{\partial W_1}{\partial K_1}=-\frac{p}{q}\cdot\frac{\partial W_1}{\partial K_1}+\frac{\partial F}{\partial W_1}\cdot\frac{\partial W_1}{\partial K_1}\\
&=\left(\frac{\partial F}{\partial W_1}-\frac{p}{q}\right)\cdot\frac{\partial W_1}{\partial K_1}=\left(2\frac{p}{q}-\frac{p}{q}\right)\cdot\frac{\partial W_1}{\partial K_1}=\frac{p}{q}\cdot\frac{\partial W_1}{\partial K_1}>0\\
\frac{\partial F}{\partial L_1} &= \frac{\partial F}{\partial E_N}\cdot\frac{\partial E_N}{\partial L_1}+\frac{\partial F}{\partial W_1}\cdot\frac{\partial W_1}{\partial L_1}=\frac{\partial F}{\partial E_N}\cdot\frac{\partial E_N}{\partial W_1}\cdot\frac{\partial W_1}{\partial L_1}\\
&=\frac{\partial F}{\partial E_N}\cdot(-1)\cdot\frac{\partial W_1}{\partial L_1}+\frac{\partial F}{\partial W_1}\cdot\frac{\partial W_1}{\partial L_1}=-\frac{p}{q}\cdot\frac{\partial W_1}{\partial L_1}+\frac{\partial F}{\partial W_1}\cdot\frac{\partial W_1}{\partial L_1}\\
&=\left(\frac{\partial F}{\partial W_1}-\frac{p}{q}\right)\cdot\frac{\partial W_1}{\partial L_1}=\left(2\frac{p}{q}-\frac{p}{q}\right)\cdot\frac{\partial W_1}{\partial L_1}=\frac{p}{q}\cdot\frac{\partial W_1}{\partial L_1}>0
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial F}{\partial K_2}&=\frac{\partial F}{\partial E_N}\cdot\frac{\partial E_N}{\partial W_2}\cdot\frac{\partial W_2}{\partial K_2}=\frac{p}{q}\cdot(-1)\cdot\frac{\partial W_2}{\partial K_2}=-\frac{p}{q}\cdot\frac{\partial W_2}{\partial K_2}<0\\
\frac{\partial F}{\partial L_2}&=\frac{\partial F}{\partial E_N}\cdot\frac{\partial E_N}{\partial L_2}\cdot\frac{\partial W_2}{\partial L_2}=\frac{p}{q}\cdot(-1)\cdot\frac{\partial W_2}{\partial L_2}=-\frac{p}{q}\cdot\frac{\partial W_2}{\partial L_2}<0
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial F}{\partial K_3}&=\frac{\partial F}{\partial W_3}\cdot\frac{\partial W_3}{\partial K_3}=\frac{\partial F}{\partial E_N}\cdot\frac{\partial E_N}{\partial W_3}\cdot\frac{\partial W_3}{\partial K_3}=\frac{p}{q}\cdot\frac{\partial W_3}{\partial K_3}>0\\
\frac{\partial F}{\partial L_3}&=\frac{\partial F}{\partial W_3}\cdot\frac{\partial W_3}{\partial L_3}=\frac{\partial F}{\partial E_N}\cdot\frac{\partial E_N}{\partial W_3}\cdot\frac{\partial W_3}{\partial L_3}=\frac{p}{q}\cdot\frac{\partial W_3}{\partial L_3}>0
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial F}{\partial K_4}&=\frac{\partial F}{\partial E}\cdot\frac{\partial E}{\partial K_4}=\frac{\partial F}{\partial E_N}\cdot\frac{\partial E_N}{\partial E}\cdot\frac{\partial E}{\partial K_4}=\frac{p}{q}\cdot\frac{\partial E}{\partial K_4}>0\\
\frac{\partial F}{\partial L_4}&=\frac{\partial F}{\partial E}\cdot\frac{\partial E}{\partial L_4}=\frac{\partial F}{\partial E_N}\cdot\frac{\partial E_N}{\partial E}\cdot\frac{\partial E}{\partial L_4}=\frac{p}{q}\cdot\frac{\partial E}{\partial L_4}>0
\end{aligned}$$

So

$$\frac{\partial F}{\partial K_2}<0;\qquad \frac{\partial F}{\partial L_2}<0 \tag{7.25}$$

and

$$\frac{\partial F}{\partial K_1}>0,\quad \frac{\partial F}{\partial L_1}>0;\quad \frac{\partial F}{\partial K_3}>0,\quad \frac{\partial F}{\partial L_3}>0;\quad \frac{\partial F}{\partial K_4}>0,\quad \frac{\partial F}{\partial L_4}>0. \tag{7.26}$$

References

[1] D. M. Rosenberg, S. Berkes, R. E. Bodaly, C. A. Kelly, & J. W. M. Rud. Large-scale impacts of hydroelectric development. *Environmental Reviews*. V. 5. p. 27–54. 1997.

[2] W. Karush. Minima of functions of several variables with inequalities as side constraints. M.Sc. Dissertation. Dept. of Mathematics, Univ. of Chicago. Chicago, Illinois. 1939.

[3] H. W. Kuhn & A. W. Tucker. Nonlinear programming. In: **Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability**. Berkeley, Calif.: University of California Press, 1951. p. 481–492. Http://projecteuclid.org/euclid.bsmsp/1200500249.

[4] D. Liberzon. **Calculus of variations and optimal control theory: a concise introduction**. Princeton, NJ: Princeton University Press, Dec 2011 2011.

[5] M. Z. Jacobson & M. A. Delucchi. Providing all global energy with wind, water, and solar power, part i: Technologies, energy resources, quantities and areas of infrastructure, and materials. *Energy Policy*. 2010.

[6] S. Lassaux, R. Renzoni, & A. Germain. Life cycle assessment of water: From the pumping station to the wastewater treatment plant. *International Journal of Life Cycle Assessment*. V. 12. p. 118–126. 2007.

[7] A. S. Stillwell, D. C. Hoppock, & M. E. Webber. Energy recovery from wastewater treatment plants in the united states: a case study of the energy-water nexus. *Sustainability*. V. 2. p. 945–962. 2010.

[8] H. Cooley & R. Wilkinson. Implications of future water supply sources for energy demands. *In WateReuse Resesarch Foundation*. 2012.

[9] O. V. Araújo. Um modelo dinâmico para integração da gestão da cadeia de abastecimento sob incerteza. PhD Thesis. Programa de Pós-Graduação em Engenharia de Produção (PPGEP) da Universidade Federal de Pernambuco. Recife. december 2008.

[10] M. d. M. e. E. MME. Resultados preliminares do balanço energético naciona. Brasília. Report. 2009. Available at: http://www.mme.gov.br.

[11] J. Conti & P. Holtberg. International energy outlook. Energy International Agency. Technical Report. 2011.

[12] Dale, Alexander T.. The Sustainable Water-Energy Nexus: Life-Cycle Impacts and Feasibility of Regional Energy and Water Supply Scenarios. D.Sc. Theses. B.S. in Engineering Physics, Univ. of Pittsburgh.Pittsburgh. 2013.

[13] M. A. Delucchi & M. Z. Jacobson. Providing all global energy with wind, water, and solar power, part ii: Reliability, system and transmission costs, and policies. *Energy Policy*. V. 39. p. 1170–1190. 2011.

[14] S. Pfister, D. Saner, & A. Koehler. The environmental relevance of freshwater consumption in global power production. *The International Journal of Life Cycle Assessment*. V. 16. p. 580–591. 2011.

[15] B. G. Ridoutt & S. Pfister. A revised approach to water footprinting to make transparent the impacts of consumption and production on global freshwater scarcity. *Global Environmental Change*. V. 20. p. 113–120. 2010.

[16] J. L. Eimers & K. O. Dennen. Usgs energy and water programs: Overview and student opportunities. *Journal of Contemporary Water Research & Education*. V. 143. p. 49–55. 2009.

[17] G. Sehlke & P. Wichlacz. U.s. department of energy's role in the energy-water nexus. *Journal of Contemporary Water Research & Education*. V. 143. p. 56–62. 2009.

[18] A. S. Stillwell. Energy-water nexus in texas. *University of Texas at Austin*. 2009.

[19] A. De La Fuente. **Mathematical Methods and Models for Economists**. Cambrige, UK: Cambridge University Press, Jan 28 2000. 835 pages.

[20] C. A. B. B. Department of Energy. In energy information administration. 2012.

[21] M. Athans & P. L. Falb. **Optimal Control: An Introduction to the Theory and its Applications**. New York: McGraw-Hill Book Company, 1966. 879 pages.

[22] R. Vinter. **Optimal Control**. Boston: Birkhauser, 2000.

[23] W. Vogt, M. Mickle, & H. Aldermeshian. A dynamic leontief model for a productive system. *Proceedings of the IEEE*. V. 63. N. 3. p. 438–443. 1975.

[24] A. Stamford da Silva & F. M. Campello de Souza. The economics of water resources for the generation of electricity and other uses. *Annals of Operations Research*. V. 154. p. 41–61. 2008.

[25] K. T. Santiago, F. M. Campello de Souza, & D. d. C. Bezerra. A strong argument for using non-commodities to generate electricity. *Energy Economics*. V. 43. p. 34–40. 2014.

[26] W. W. Leontief. **Input-Output Economics**, 2ª ed. New York: Oxford University Press, 1986.

[27] B. Barker. Running dry at the power plant. *EPRIJournal Summer*. p. 26–35. 2007.

[28] A. C. Chiang. **Elements of Dynamic Optimization**. McGraw-Hill, 1992.

[29] M. D. Intriligator. **Mathematical Optimization and Economic Theory**. Society for Industrial and Applied Mathematics, 2002.

[30] R. Dorfman. An economic interpretation of optimal control theory.*The American Economic Review*. 1969.

[31] P. Dasgupta & G. M. Heal. The optimal depletion of exhaustible resources. *The Review of Economic Studies*. p. 3–28. 1974. Symposium.

[32] J. H. v. Geldrop & C. A. A. M. Withagen. General equilibrium in an economy with exhaustible resources and an unbounded horizon. *Journal of Economic Dynamics And Control*. V. 5. N. 18. p. 1011–1035. 1994.

[33] F. M. Campello de Souza. Introdução do aquecimento solar na matriz energética. 1997. Projeto de Pesquisa CNPq. mimeo.

[34] F. M. Campello de Souza. **Controle Ótimo**. Recife, Pernambuco, Brasil: Fernando Menezes Campello de Souza, 2014. Livro em preparação.

[35] Y. H. Farzin. Optimal saving policy for exhaustible resource economies. *Journal of Development Economics*. V. 58. p. 149–184. 1999.

[36] H. Hotelling. The economics of exhaustible resources. *Journal of Political Economy*. V. 39. N. 2. p. 124–142. April 1931.

[37] M. A. Adelman. Modeling world oil supply. *Energy Journal*. V. 14. N. 1. p. 1–32. 1993.

[38] J. Macknick, R. Newmark, H. Garvin, & K. C. Hallett. A review of operational water consumption and withdrawal factors for electricity generating technologies. National Renewable Energy Laboratory. Golden, Colorado. Technical Report NREL/TP – 6A20 – 50900. March 2011. NREL is a national laboratory of the U.S. Department of Energy, Office of Energy Efficiency & Renewable Energy, operated by the Alliance for Sustainable Energy, LLC.

[39] WWAPW. W. W. A. Programmme). The united nations world water development report 4: Managing water under uncertainty and risk, united nations educational scientific and cultural organization. *Paris*. 2012.

[40] textscE. Mielke, L. D. Anadon, & V. Narayanamurti. Water consumption of energy resource extraction, processing and conversion, belfer center for international affairs. *Harvard Kennedy School of government, Cambridge, United States*. October 2010. Energy Technology Innovation Policy Discussion Paper No. 2010-15

[41] textscJ. Schornagel, F. Niele, E. Worrell, & M. Böggemann. Water accounting for (agro) industrial operations and its application to energy pathways. *Resources, Conservation and Recycling, Elsevier, Amsterdam*. V. Vol. 61. p. 1–15. 2012.

[42] U. N. F. UNFAO & A. Organization). The wealth of waste: The economics of wastewater use in agriculture. *UN FAO, Rome*. 2010.

[43] ——. Aquastat database. *UN FAO, Rome*. p. http://www.fao.org/nr/water/aquastat/main/index.stm. 2012.

[44] I. I. P. on Climate Change). Climate change and water. *Technical Paper, IPCC, Geneva*. 2008.

[45] I. I. E. Agency). Golden rules for a golden age of gas: World energy outlook special report on unconventional gas. *OECD/IEA, Paris*. 2012.

[46] S. B. Roy, L. Chen, E. Girvetz, E. Maurer, W. B. Mills, & T. M. Grieb. Sustainability of projected water demands under future climate change scenarios. *TetraTech Inc.* 2010.

[47] P. Gleick. Water and energy. *Annual Review of Energy and Environment.* V. Vol. 36, No. 3. p. 267–299. 1994.

[48] I. Shiklomanov. Water in crisis: A guide to the world's fresh water resources. World Fresh Water Resources. *Gleick, P. (ed.), Oxford University Press, New York* p. 13–24. 1993.

[49] B. C. Bates, Z. W. Kundzewicz, S. Wu, & J. P. Palutikof. Climate change and water. *Technical Paper of the Intergovernmental Panel on Climate Change. Geneva, Switzerland: IPCC.* 2008.

[50] A. Xepapadeas. Economic growth and the environment. K.-G. Mäler & J. R. Vincent, Eds. In: **Handbook of Environmental Economics**. ser. Handbook of Environmental Economics. Elsevier, January 2005. V. 3. cap. 23. p. 1219–1271. [Online]. Disponível: `http://ideas.repec.org/h/eee/envchp/3-23.html`

[51] J. F. Kenny, N. L. Barber, S. S. Hutson, K. S. Linsey, J. K. Lovelace, & M. A. Maupin. Estimate use of water in the united states in 2005. *USGS.* 2005.

[52] A. Voinov & H. Cardwell. Energy-water nexus: why should we care? *Journal of Contemporary Water Research and Education.* p. 143. 2009.

[53] P. J. Crabbe. The contribution of l.c.gray to the economic theory of exhaustible natural resources and its roots in the history of economic thought. *Journal of Environmental Economics and Management.* V. 3. p. 95–220. September 1983.

[54] L. H. Endress & J. A. Roumasset. Golden rules for sustainable resource management. *The Economic Record.* V. 70. N. 210. p. 267–277. September 1994.

[55] J. A. Krautkraemer. Nonrenewable resource scarcity. *Journal of Economic Literature.* V. XXXVI. p. 2065–2107. Dec. 1998.

[56] D. L. Krulce, J. A. Roumasset, & T. Wilton. Optimal management of renewable and replaceable resource: the case of coastal groundwater. *American Journal of Agricultural Economics.* V. 79. p. 1218–1228. Nov. 1997.

[57] W. Nordhaus. The allocation of energy resource. *Brookings Papers on Economic Activity*. V. 3. 1973.

[58] S. W. Salant. The economics of natural resource extraction: a primer for development economists. *The World Bank Observer*. Feb. 1995. Washington.

[59] R. Solow. Intergenerational equity and exhaustible resources. *Review of Economic Studies*. p. 29–45. 1974. Symposium.

[60] J. Stiglitz. Growth with exhaustible natural resources: efficient and optimal growth. *Review of Economic Studies*. p. 123–137. 1974. Symposium.

[61] R. A. Young. Measuring economics benefits of the policies and investment for water. *World Bank Technical Paper*. N. 338. September 1996.

[62] J. M. Chermak & R. H. Patrick. Comparing tests of the theory of exhaustible resources. *Resource and Energy Economics*. V. 24. p. 301–325. 2002.

[63] M. Macleod, D. Moran, & I. Spencer. Counting the cost of water use in hydroelectric generation in scotland. *Energy Policy*. V. 34. N. 15. p. 2048–2059. 2006.

[64] B. Michalland. Bi-objective dynamic programming for trading off hydropower and irrigation. *Applied Mathematics and Computation*. V. 88. p. 53–76. 1997.

[65] B. Chatterjee, R. E. Howitt, & R. J. Sexton. The optimal joint provision of water for irrigation and hydropower. *Journal of Environmental Economics and Management*. V. 36. p. 295–313. November 1998.

[66] L. S. Pontryagin, V. G. Boltyanski, R. V. Gamkrelidze, & E. F. Mishenko. **The mathematical theory of optimal processes**. L. W. Neustadt, Ed. New York: John Wiley & Sons (Interscience Publishers), 1962.

[67] J. L. Eimers & K. O. Dennen. Usgs energy and water programs: Overview and student opportunities. *Journal of Contemporary Water Research & Education*. V. 143. p. 49–55. 2009.

[68] CHIANG, Alpha C. Elements of Dynamics Optimization. Mc Graw - Hill, 1992. P. 159-313

[69] IYANAGA, S. and KAWADA, Y. (Eds.) Pontrjagin's Maximum Principle. 88C in Encyclopedic Dictionary of Mathematics. Cambridge, MA: MITPress, 1980. p. 295-296

[70] DORFMAN, Robert. An Economic Interpretation of Optimal Control Theory. The American Economic Review. Dec. 1969. LIX v., n. 5, p.817-831

[71] DINAR, Ariel, ROSEGRANT, Mark W. and MEINZEN-DICK, Ruth. Water allocation mechanisms: principles and examples. A publication of World Bank, 1998.

[72] YOUNG, Robert A. Measuring Economics Benefits of the Policies and Investment for Water. September 1996. World Bank Technical Paper. n. 338.

[73] THEIS, Ivo M. Limites Energéticos do Desenvolvimento, Blumenau: Ed. da FURB, 1996.

[74] EDMONDS, Jae e REILLY, John M. Global Energy Assessing the Future. New York-USA: Oxford University Press, 1985. Cap. 1-5. p. 3-64.

[75] GOLDEMBERG, José. Energia no Brasil. Rio de Janeiro: LTC, 1979.

[76] ENDRESS, Lee H. and ROUMASSET, James A. Golden Rules for Sustainable Resource Management. The Economic Record. September 1994. 70 v., n. 210, p. 267-277

[77] Torcellini, P., N. Long and R. Judkoff (2003), Consumptive Water Use for US Power Production, National Renewable Energy Laboratory, Golden, United States.

[78] CAMPELLO DE SOUZA, Fernando. Sistemas de Controle Auto Adaptativos. Notas de Aula do curso de especialização em Sistemas de Controle (Section Spéciale d'Automatique). Toulouse, França : École Nationale Superieure d'Electronique, d'Informatique et d'Hydraulique de Toulouse (ENSEEIHT), 1970.

[79] CAMPELLO DE SOUZA, Fernando. Introdução do Aquecimento Solar na Matriz Energética. Recife, Dezembro de 2000. Projeto de Pesquisa CNPq Referência Processo: 300832/79-6.

[80] QUESNAY, FRANÇOIS. Tableau économique (Physiocratie, ou Constitution naturelle du Gouvernement économique d'un royaume agricole). *Dupont de Nemours.*Leyde. 1758.

[81] DORF, RICHARD C. AND BISHOP, ROBERT H.. Modern Control Systems. *Prentice Hall.*New Jersey. 2011.

[82] OGATA, KATSUHIKO. Modern Control Engineering. *Prentice Hall.*Boston. 2010.

[83] Elgerd, Olle I.. Control Systems Theory. *McGraw-Hill.*New York. 1967.

[84] Chen, Chi-Tsong. Linear System Theory and Design. *Oxford University Press.* 2012.

[85] Govil, Manish and Proth, Jean-Marie. Supply Chain Design and Management: Strategic and Tactical Perspectives. *Academic Press.*San Diego. 2002.

[86] Simchi-Levi, David and Kaminsky, Philip and Simchi-Levi, Edith. Designing and Managing the Supply Chain: Concepts, Strategies, and Cases. *Mc-Graw-Hill.*Ohio. 2007.

[87] Hastings, S. E. and Brucker, S. M.. Microcomputer based input-output modeling: Applications to economic development. *Westview Press.*Colorado. 1993.

[88] Raa, Thijs ten. The Economics of Input–Output Analysis. *Cambridge University Press.*Cambridge, UK. 2005.

[89] Raa, Thijs ten. Input-Output Economics: Theory and Applications; Featuring Asian Economies. *World Scientific.*Singapore. 2009.

[90] Dias, Fernando de Mendonça and Bezerra, David Ricardo Colaço and Ramos, Francisco de Sousa. Conflito no uso da áagua: a importâancia da ação coordenada das agências reguladoras. *Revista Econômica do Nordeste.* V. 31. p. 798–808. 2000.

[91] Dias, Fernando de Mendonça and Bezerra, David Ricardo Colaço and Ramos, Francisco de Sousa. A ação das agências reguladoras no uso eficiente da água para irrigação e produção de energia. *Revista Econômica do Nordeste.* V. 32. p. 783–791. 2001.

[92] Vogt, William and Mickle, Marlin and Aldermeshian, Hrair. A dynamic Leontief Model for a productive system. *Proceedings of the IEEE.* V. 63. p. 438–443. 1975.

[93] Molle, François and Cardier, Eric. Manual do Pequeno Açude. *SUDENE.*Recife. 1992.